Screening by Mode of Trade[*]

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Abstract

This paper endogenizes a monopolist's choice between selling and renting in a non-anonymous durable goods setting with short-term commitment, by allowing for contracts that determine the good's allocation not only at the beginning but also at the end of a given period. We show that the revenue-maximizing menu of contracts features *screening by mode of trade* when future trade is subject to frictions and the monopolist is more patient than consumers. Selling to high types while renting to low types, allows the monopolist to defer part of his compensation in form of a reduction of consumers' future information rents while lowering the allocational costs of ordinary, intertemporal screening.

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1 Introduction

The loss of market power resulting from a monopolist's inability to commit to future terms of trade, has been a dominant theme of the durable goods literature, covering both, selling frameworks (Coase, 1972) and renting frameworks (Hart and Tirole, 1988). What has seemingly gone unnoticed is the fact that requiring the monopolist to *either* sell *or* rent imposes restrictions on the set of implementable allocations which could have similar consequences as those implied by his lack of commitment. The main objective of this article is to investigate the conditions under which *screening by mode of trade*, i.e. selling to some consumers while renting to others, can improve screening in a durable goods monopoly.

Examples for the coexistence of rentals and sales are numerous and range from housing and industrial machinery to cars and musical instruments. With the emergence of e-commerce, the marketing of electronic content such as e-books, movies, or songs, both as streaming- and download-versions has become common practice. Although alternative reasons such as limited budgets or preference uncertainty may motivate the supply of a rental option, the analysis of its effect on the persistence of informational asymmetries is a crucial element for our understanding of the determinants of market power in durable goods markets.

We investigate the possibility of screening by mode of trade in the canonical durable goods model of Hart and Tirole (1988), outlined in Section 2. A single, risk-neutral consumer has unit demand for a durable product during two periods. The consumer's per-period valuation of the product can take two values, is constant across time, and constitutes the consumer's private information, i.e. his *type*. The product is provided by a risk-neutral, monopolistic supplier with zero cost. In every period, the supplier can offer a menu of *short-term contracts*. A short-term contract specifies a monetary transfer, made during that period, as well as the probabilities of allocating the product to the consumer at the beginning of the period and of re-allocating the product to the supplier at the end. The inclusion of the product's probability of re-allocation allows us to fully endogenize the supplier's choice between renting and selling. Our focus on short-term contracting is in line with the literature's standard assumption that the supplier lacks (long-term) commitment in that transfers and (re)allocation-probabilities can only be specified for the current period. Our model offers new insights about the nature of contracting for, what Hart and Tirole denominate as, the *soft supplier* case, referring to a supplier whose prior expectations about the consumer's type fall below the ratio of valuations.

Rationalizing the coexistence of selling and renting turns out to require two diversions from the standard model. First, we assume that future trade opportunities are not guaranteed but become obstructed with an exogeneous probability. This assumption is motivated by the observation that in many instances renters wish they would have bought a product because its supplier is no longer allowed to offer it to them legally, has found another client, has gone out of business, or demands the product for himself.[1] Second, we allow for the possibility of heterogeneous discounting by assuming that the monopolist is patient, in that he discounts future payoffs less strongly than the consumer. Arguably, the case of a patient supplier is relevant, because firms may have access to cheaper credit (Hirshleifer, 1958), or because consumers might be present-biased (O'Donoghue and Rabin, 2015).[2]

In Section 3, we consider as a benchmark the commitment case where the supplier can

[1]The 2019 US trade embargo against Chinese firms has impeded Google from continuing to license Huawai for the use of Google-apps on Huawei-smartphones. There are numerous instances where artists have removed their music from media platforms following copyright-disputes, making individual songs unavailable for those consumers who opted for streaming rather than download (see e.g. https://time.com/3554438/taylor-swift-spotify/). Disruptions of rental-agreements are also frequent in housing markets, and there is evidence that the share of rentals increases in the efficiency of courts (Casas-Arce and Saiz, 2010).

[2]Although heterogeneous time preferences are commonly assumed in related models of bargaining with asymmetric information (Fudenberg et al., 1983; Sobel and Takahashi, 1983), the durable goods literature has largely focused on the case of homogeneous discounting. Relaxing this assumption can reveal important features of the optimal trading mechanism, e.g. the sub-optimality of price-posting (Beccuti and Möller, 2018). Heterogeneous discounting has been employed to explain a monopolist's choice of product durability (e.g. Barro, 1972) and the emergence of behavior-based price discrimination in a *non-durable* goods framework with long-term commitment (Bikhchandani and McCardle, 2012).

offer a menu of *long-term contracts*, specifying transfers and (re)allocation probabilities for *both* periods. The soft supplier's optimal menu of long-term contracts pools types by allocating the product to the consumer in both periods in exchange for a transfer from the consumer to the supplier executed in period 2. In reality, such an outcome is frequently implemented in form of sale contracts with deferred payments which insure against the absence of future trade opportunities while making optimal use of differences in the parties' discount factors.

Under short-term contracting, parties cannot commit to future transfers so that sales with deferred payment are no longer feasible. It is not uncommon, for instance, that deferred payments are ruled out by large financial risks, arising either from the sheer size of the transaction (e.g. housing markets) or the lack of legal enforceability (e.g. overseas or online markets). Without the possibility of deferred payment a trade-off arises. While only selling can guarantee that, as in the benchmark, the product is allocated to the consumer in both periods, only renting offers the possibility that (part of) the supplier's compensation can be postponed. In Section 5, we start our analysis of short-term contracting by considering the case where the monopolist is restricted to use a *single mode of trade*, i.e. *either* selling *or* renting. A straight forward implication of the above trade-off is that the monopolist prefers renting when the likelihood of future trade opportunities is high whereas selling is optimal when this likelihood is low.

Besides making our results comparable to the existing literature, the analysis in Section 5 reveals an important element for our understanding of the optimality of screening by mode of trade. We show that, under short-term contracting, a soft supplier may benefit from the separation of types because it allows him to "defer" at least a part of his payment, in form of a reduction in the consumer's future information rents. When the supplier is restricted to a single mode of trade, separation comes at the cost of excluding the low type in period 1, which is why separation is still dominated by pooling when the monopolist's prior is sufficiently low.

Surprisingly, when the monopolist can separate types by offering a menu containing a selling *and* a renting contract, this conclusion may no longer be valid. In Section 6, we characterize the optimal menu of short-term contracts when the monopolist can employ both modes of trade. Our main result shows that the separation of types can be optimal for arbitrarily low priors. The reason is that screening by mode of trade, i.e. selling to high types while renting to low types, eliminates the need to exclude the low type from first period trade. Screening is still costly because it requires the monopolist to offer the "wrong mode of trade" to one of the two types. However, when the monopolist is indifferent between selling and renting, which happens when the likelihood of future trade opportunities takes intermediate values, screening comes practically for free.

The main insight of our analysis can therefore be summarized as follows. For a patient supplier facing an uncertain trade horizon, screening by mode of trade arises as a consequence of the supplier's lack of commitment to long term contracts, which introduces a trade-off between allocative efficiency (selling) and deferred payment (renting). Screening by mode of trade offers an alternative form of deferred payment as it induces high types to give up their information rents in the future without the need to exclude low types from trade today. Our discovery of screening by mode of trade for a soft supplier complements Hart and Tirole's finding of semi-separation for a tough supplier, as special features of contracting in the absence of long-term contracts.

Related literature. Although our theory is set in a durable goods framework, it relates, more generally, to the literature on dynamic adverse selection, initiated by Freixas et al. (1985) and Laffont and Tirole (1987, 1988). An important insight of this literature is that intertemporal screening by time might be substituted by intra-temporal screening by menu (Wang, 1998). The durable goods literature has employed a similar idea to an *anonymous* market with differentiated varieties (Kühn and Padilla (1996); Kühn (1998); Takeyama, 2002; Hahn, 2006; Inderst, 2008). Kühn and Padilla's work on the simultaneous supply of a durable and a non-durable substitute bears some similarity to screening by mode of trade. However, our theory differs in its approach (general contracting) and focus (non-anonymous markets) and demonstrates that the mere choice between buying and renting of a *single* product-variety can be sufficient to obtain immediate separation of consumer types.

The conditions under which screening by mode of trade prevails in our model – uncertain trade opportunities and heterogeneous time preferences – are realistic features of many markets and constitute regular, albeit alternate assumptions of the related literature on bargaining (e.g. Rubinstein and Wolinsky, 1985; Binmore et al., 1986; Fudenberg et al., 1983; Sobel and Takahashi, 1983). While in our setting the likelihood of a future trade opportunity is exogeneous, first steps towards an endogenously determined trade horizon have been made recently both for a selling framework (Board and Pycia, 2014) and a rental setting (Gerardi and Maestri, 2018), by enabling one of the parties to abandon the trade-relationship permanently.

While selling induces a time-invariant consumption pattern, renting entails the possibility that consumption becomes "renegotiated" in the future. Laffont and Tirole (1990) consider a two-period renting framework with a divisible good, where a monopolist can offer long-term contracts that are subject to renegotiation. They show that the contract designed for the high type induces the same (efficient) consumption level in both periods while the contract designed for the low type becomes renegotiated.[3] Although this pattern bears some similarity to screening by mode of trade, an important difference is that the low type's consumption becomes renegotiated *upwards* whereas in our setting trade with the low type is first efficient and then moves *downwards*. It is the deferral of allocative efficiency into the future, which explains the power of the mode of trade as a screening device.

Our result that combining selling with renting may reduce distortions arising from

[3] Maestri (2017) finds that in the limiting case of an infinite horizon and no discounting, the low type's renegotiated contract becomes approximately efficient, thereby eradicating the monopolist's ability to screen customers. See also Strulovici (2017).

asymmetric information is reminiscent of the idea that leasing can mitigate the lemons problem in resale markets (Hendel and Lizzeri, 2002; Johnson and Waldman, 2003). Leasing contracts differ from renting contracts in that they entail the supplier's commitment to a future (selling) price. When lessees obtain private information about their product's (depreciated) quality, the associated option value is increasing in their valuation of quality. Hence, while both renting and leasing might serve a screening purpose in a durable goods setting, the conditions under which they emerge as an alternative to buying are markedly different.

Finally, it is important to note that, due to the dynamic nature of the durable goods problem, renting differs from buying not only by offering a "low quality" alternative, familiar from the literature on static screening (Mussa and Rosen, 1978; Matthews and Moore, 1987), but also by exposing the consumer to the future consequences of revealing information about his type (ratchet effect).

2 Model

We consider a monopolistic supplier of a non-divisible, durable product, facing a single consumer during two periods.[4] The supplier's costs are normalized to zero. In each period, the consumer has unit demand. The consumer's per-period valuation of the supplier's product, θ, is strictly positive and constant over time.

Information. θ can take two values and constitutes the consumer's private information which is why we denote it as the consumer's type, indexed by $i \in \{L, H\}$. With probability $\beta \in (0, 1)$ the consumer's valuation is high, $\theta = \theta^H$, whereas with probability $1 - \beta$ the consumer's valuation is low, $\theta = \theta^L < \theta^H$. We call β the supplier's prior belief and abbreviate notation by defining $\Delta\theta \equiv \theta^H - \theta^L$. Most of our analysis focuses on the case

[4] A discussion of the effects of extending our model to allow for more than two periods is postponed until the Conclusion. Our model allows for the interpretation of a continuum of *non-anonymous* consumers. Importantly, the same (set of) consumer(s) is present in all periods. This distinguishes us from the literature studying the effects of consumers arriving over time (Conlisk et al., 1984; Board, 2008; Deb and Said, 2015; Garrett, 2016).

of a *soft* supplier by assuming that $\beta < \frac{\theta^L}{\theta^H}$.[5]

Payoffs. If in any given period, the consumer makes a transfer t to the supplier, the supplier's instantaneous payoff is given by his revenue t. If the product is allocated to the consumer during that period, the consumer's instantaneous payoff is $\theta - t$, otherwise the consumer's payoff is $-t$. The supplier and the consumer discount future payoffs with discount factors $\delta_S \in (0,1)$ and $\delta_C \in (0,1)$, respectively. We thus allow for heterogeneous discounting but restrict attention to the interesting case by making the following

Assumption 1 (Patience)**.** *The supplier is patient, i.e.* $\delta_S > \delta_C$[6].

Contracts. In accordance with the more recent durable goods literature, we assume that commitment is limited in that contracting cannot reach beyond the current period. More specifically, in every period, the supplier can offer a menu of *short-term contracts.* A short term contract (d, r, t) specifies: a probability $d \in [0,1]$ with which the supplier's product is delivered to the consumer at the beginning of the period; a probability $r \in [0,1]$ with which the product must be returned to the supplier at the end of the period; and a transfer $t \geq 0$ from the consumer to the supplier executable during that period. *Long-term contracts*, specifying not only current but also future actions are explicitly ruled out and analyzed separately in the context of the commitment benchmark in Section 3. The requirement that transfers be non-negative reflects the assumption that the supplier cannot lend money to the consumer.[7] Restricting attention to deterministic transfers is without loss of generality because utilities are quasi-linear.

Timing. In period 1, the monopolist offers a menu of short-term contracts. The consumer accepts one or none of these contracts. If the consumer accepts a contract, the contract is executed, determining the product's allocation and the transfer from the

[5]The analysis of the complementary case of a tough supplier is subject of Section 7.

[6] It will become clear that the analysis of the remaining case where $\delta_C \geq \delta_S$ is trivial and it is therefore omitted.

[7]As we will see, this assumption is innocuous for our analysis of short-term contracting, but endows the commitment benchmark with a well-defined solution.

consumer to the supplier during period 1. We denote the product as *sold* if it is delivered without being returned. Rejection moves the game to period 2 without any trade or transfer occurring. In period 2, conditional on the product not having been sold, the monopolist offers a new – potentially different– menu of short term contracts from which the consumer may choose. To capture the idea that renting jeopardizes potential future gains from trade that could have been realized through a sale, we make the following assumption:

Assumption 2 (Trade frictions)**.** *In period* 2*, trade between the two parties, both in form of product- or money-transfers, is obstructed with probability* $1 - \phi \in (0, 1)$.

If trade becomes obstructed, then any contract between the two parties becomes void, and neither the supplier's product nor money can be exchanged. This assumption captures the idea that a sale provides a particular form of "commitment" that is not necessarily achieved through long-term contracting.[8] In particular, in the presence of trade-frictions, a sale in period 1 is the only way to guarantee that the product is allocated to the consumer in period 2. In the limit, where $\phi \to 1$ and $\delta_S \to \delta_C$ our setting converges to a two-period version of Hart and Tirole (1988).

Strategies. In period 2, our setting is identical to a static screening problem and the supplier's revenue is maximized by simple *price-posting*, implementable via the menu of contracts $\{(1, 0, p), (0, 0, 0)\}$. More specifically, it is well established that, without loss of generality, the supplier's behavior in period 2 can be fully described by a price offer $p \in \{\theta^L, \theta^H\}$ which a consumer of type $i \in \{L, H\}$ accepts if and only if $p \leq \theta^i$. The consumer's payoff is given by $\max(0, \theta^i - p)$ whereas the supplier's revenue is $R_i(p) = p$ if $p \leq \theta^i$ and $R_i(p) = 0$ otherwise. In period 1, the supplier's problem constitutes a mechanism design problem with limited commitment. The supplier can commit to contracts (d, r, t) governing period 1 but cannot commit to his price offer p for period

[8] Licensing of Google-apps for Huawei smartphones was interrupted even though the firms' agreements extended beyond the trade-embargo inflicted by the US government.

2. Most generally, the supplier's strategy can be modeled as the choice of a mechanism (M, c), consisting of a message space M and a contract choice function c. Any mechanism induces a game in which first, the consumer selects a probability distribution q over messages $m \in M$ resulting in the contract choice $c(m) = (d(m), r(m), t(m))$, second, the supplier updates his belief about the consumer's type to $\tilde{\beta}(m)$ based on his observation of m, and, third, the supplier chooses a price p.[9] If the consumer chooses not to participate in the supplier's mechanism, no contract is selected, i.e. the product is not allocated to the consumer in period 1 and the game moves directly to period 2.

Equilibrium. We use Perfect Bayesian Equilibrium (PBE) as solution concept which requires that for any choice of mechanism (M, c), the consumer's communication strategy q maximizes his expected payoff, the supplier updates his belief about the consumer's type $\tilde{\beta}$ based on Bayes rule whenever feasible, and the supplier's price offer p maximizes his expected second period revenue $\tilde{\beta}R_H(p) + (1 - \tilde{\beta})R_L(p)$ given his updated belief. As Perfect Bayesian equilibrium puts no restrictions on off-equilibrium beliefs, we can assume that, following the consumer's non-participation in the mechanism, the supplier believes the consumer to be of a high type. Given this belief, the consumer's reservation payoff equals zero, independently of his type.

Revelation principle (modified). We are interested in determining a mechanism that maximizes the supplier's expected payoff. The following lemma shows that, within the set of so called "incentive efficient mechanisms," inducing Pareto-efficient payoffs, there is no loss of generality in restricting attention to a simple class of "direct" mechanisms, in which the consumer chooses from a menu consisting of only two contracts and reveals his true type with positive probability.

[9] Implicit in our formulation of the model as a contracting problem is the assumption that the supplier can observe the consumer's choice of contract. A more general approach would consider the consumer's message as the input of a communication device whose output is observed by the monopolist (see Bester and Strausz, 2007 and Doval and Skreta, 2019a). We discuss the possible effects of allowing for such mechanisms with noisy communication in Section 6.

Lemma 1. *Suppose that a mechanism* (M, c) *and resulting PBE* $(q, \tilde{\beta}, p)$ *gives the supplier and the consumer the expected payoffs* V *and* U^i*, respectively, and there exists no mechanism* (M, c') *and resulting PBE* $(q', \tilde{\beta}', p')$ *that increases the supplier's payoff to* $V' > V$ *while guaranteeing every consumer type* $i \in \{L, H\}$ *the same payoff* U^i*. Then* V *can be obtained with a mechanism with message space* $\{L, H\}$ *and in the corresponding PBE, type* $i \in \{L, H\}$ *chooses message* $m = i$ *with positive probability.*

Lemma 1 is a direct adaption of the modified revelation principle by Bester and Strausz (2001). The proof, contained in the Appendix, uses the fact that, due to the simple nature of our second period contracting problem, the dynamic mechanism design problem in period 1 is isomorphic to a static mechanism design problem with contractible actions, (d, r, t), and non-contractible actions, p. Note, however, that the modified revelation principle differs from the standard revelation principle, familiar from static contexts, in two aspects. First, it allows for mechanisms where types are misrepresented with positive probability. Second, a restriction to direct mechanisms may not be without loss of optimality, because when no optimal first period mechanism exists, a potentially existing optimal direct mechanism might be suboptimal.[10]

Having noted these reservations, in the following we restrict the monopolist's strategy in period 1 to the choice of a binary menu of contracts $\{(d_L, r_L, t_L), (d_H, r_H, t_H)\}$, and characterize the consumer's strategy by two numbers $q^L, q^H \in [0, 1]$ such that $q_L < 1$ and $q_H > 0$. The interpretation is that type $i \in \{L, H\}$ accepts the H-contract (d_H, r_H, t_H) with probability q^i while accepting the L-contract (d_L, r_L, t_L) with the complementary probability $1 - q^i$.[11] We can assume, without loss of generality, that $q^H \geq q^L$, because if this was not the case, we could simply rename contracts. The main difficulty, arising from the supplier's limited commitment, is that we cannot restrict attention to contract menus that induce full type revelation, i.e. $q^L = 0$ and $q^H = 1$, but must allow for the

[10] We are grateful to one of the referees for pointing out this existence issue.

[11] Note that we indicate contracts by subscript to facilitate distinction from the consumer's type denoted by superscript.

possibility that types are "misrepresented".

There are two notable features of our model. Firstly, our set of feasible contract menus contains as special cases the pure selling menus ($r_L = r_H = 0$) and renting menus ($r_L = r_H = 1$) considered by the existing literature. Secondly, and most importantly, a generic contract in our model cannot be replicated via the mere re-definition of an "allocation", neither as a selling nor as a renting contract. To see this, let $m \in \{L, H\}$ denote the consumer's (realized) choice of contract from the menu $\{(d_L, r_L, t_L), (d_H, r_H, t_H)\}$. Let $\tilde{\beta}_m$ be the supplier's updated belief about the consumer's type conditional on m and let $\tilde{U}^i(\tilde{\beta}_m)$ denote the consumer's second period gains from trade when his type is $i \in \{L, H\}$ (both to be determined in Section 4). In period 1, type i's expected payoff from choosing contract m can then be written as

$$U^i_m \quad = \quad d_m\theta^i - t_m + \delta_C \left\{ \underbrace{d_m(1 - r_m)}_{\text{sold}} \theta^i + \underbrace{[1 - d_m(1 - r_m)]}_{\text{unsold}} \phi \tilde{U}^i(\tilde{\beta}_m) \right\}. \tag{1}$$

Note from (1) that a selling- and a renting-contract differ in that only the former exhibits a direct, *non-informational* link between present allocation and future payoffs. More specifically, while under selling ($r_m = 0$), future payoffs depend on d_m directly, under renting ($r_m = 1$), the consumer's choice between d_L and d_H influences his future payoffs only indirectly via its effect on the supplier's updated belief $\tilde{\beta}_m$. The reason for this difference is that under selling, trade is an "absorbing state" in the language of Tirole (2016), whereas under renting trade is non-absorbing. In our approach, the return probabilities r_m can be used to fine-tune this non-informational link between present allocation and future payoffs. The supplier controls how absorbing trade is and can even make trade with one type more absorbing than with the other by choosing $r_L \neq r_H$. It is in this sense that our approach *extends* the existing renting or selling models.

3 Benchmark: Long-term contracts

In this section, we consider as a benchmark the case where the parties can commit to *long-term* contracts, specifying (re)allocation probabilities and transfers not only for the *current* but also for *future* periods. In particular, we consider the following modification to our baseline model.

In period 1, the supplier can offer a menu $\{(d_L^1, r_L^1, t_L^1, d_L^2, t_L^2), (d_H^1, r_H^1, t_H^1, d_H^2, t_H^2)\}$ of long-term contracts. Similar to a short-term contract, a long-term contract specifies the probabilities of product delivery d_m^1 and return r_m^1 as well as a transfer t_m^1 for period 1. In addition, a long-term contract also specifies a probability of product delivery d_m^2 and a transfer t_m^2 for period 2 (return probabilities being redundant). If the consumer rejects all contracts, play moves to period 2 just as under short-term contracting. If the consumer accepts a long-term contract $(d_m^1, r_m^1, t_m^1, d_m^2, t_m^2)$ then in period 2, the product's delivery is arranged and a transfer is executed as specified by the contract, *conditional on trade not becoming obstructed by market frictions.*[12] The assumption that trade-frictions cannot be overcome by contracting makes a sale ($d^1 = 1$, $r^1 = 0$) different from a repeated rental ($d^1 = 1$, $r^1 = 1$, $d^2 = 1$), both under short- and long-term contracting. It allows for a consistent comparison between the two contracting regimes.

The derivation of the supplier's optimal menu of long-term contracts is simplified by the following observation. If t_m^1 is reduced by one unit and t_m^2 is increased by $\frac{1}{\delta_C \phi}$ units then the consumer's expected discounted transfer remains the same but the supplier's revenue increases by $\phi \delta_S \frac{1}{\delta_C \phi} - 1 = \frac{\delta_S}{\delta_C} - 1$ units. Hence, for a patient supplier, any optimal long-term-contract must set $t_m^1 = 0$, i.e. all transfers must be deferred to period 2. Given this insight, the consumer's choice of a long-term contract is completely determined by

[12] Whether the transfer t_m^2 is executed conditional or unconditional on trade being unobstructed is irrelevant for our analysis of long-term contracting, because due to risk-neutrality only expected payments matter.

t^2_m and his expected discounted valuation of the induced allocation, $x_m\theta^i$, where

$$x_m \equiv d^1_m + \delta_C[d^1_m(1-r^1_m) + (1-d^1_m+d^1_m r^1_m)\phi d^2_m]. \tag{2}$$

The supplier's long-term contracting problem thus takes the familiar form of a static screening problem:

$$\max_{(x_L,t^2_L),(x_H,t^2_H)} \delta_S\phi[\beta t^2_H + (1-\beta)t^2_L] \tag{3}$$

subject to participation and incentive constraints

$$x_i\theta^i - \delta_C\phi t^2_i \geq 0, \quad i \in \{L,H\} \tag{4}$$

$$x_i\theta^i - \delta_C\phi t^2_i \geq x_m\theta^i - \delta_C\phi t^2_m, \quad i,m \in \{L,H\}, m \neq i. \tag{5}$$

The solution to this problem is standard and described in the proof of Proposition 1 contained in the Appendix. Here we state and discuss the result in the form of the following:

Proposition 1 (Commitment-Benchmark)**.** *The optimal menu of long-term contracts of a patient and soft supplier pools types by offering a single selling contract ($d^1 = 1$, $r^1 = 0$) accepted by both types and by deferring all transfers to the future ($t^1 = 0$, $t^2 = \frac{(1+\delta_C)\theta^L}{\phi\delta_C}$).*

The supplier's optimal long-term contract achieves two objectives. First, by deferring all transfers to the future, the optimal long-term contract makes use of the parties difference in discount factors, $\delta_S - \delta_C > 0$. Second, by implementing a sale in period 1, the optimal long-term contract avoids potential trade frictions and guarantees the efficient allocation of the supplier's product for *both* periods. The supplier's maximized revenue under long-term contracting is given by

$$V^{LT} = \delta_S\phi \cdot \frac{(1+\delta_C)\theta^L}{\phi\delta_C} = \frac{\delta_S}{\delta_C}(1+\delta_C)\theta^L. \tag{6}$$

As will become clear from our analysis in the next sections, no short-term contract can combine the two features of the optimal long-term contract described above, making the payoff V^{LT} unattainable. Note, however, that V^{LT} can be obtained by use of a long-term contract even when such a contract can be renegotiated in period 2. In particular, a long term contract that allocates the product to the consumer in period 1 and defers the supplier's compensation to period 2 is renegotiation-proof. Hence, all that is required to obtain V^{LT} is that parties are able to make an agreement in period 1 that binds them in period 2 unless there is mutual agreement to divert from it. In our model, this possibility is ruled out by the assumption that long-term contracts are not available.

4 Short-term contracting: The supplier's problem

In this section, we start our analysis of short-term contracting, by showing that the menu of contracts that maximizes the supplier's revenue in a Perfect Bayesian equilibrium – subsequently denoted as the supplier's *optimal menu* – can be determined as the solution of a linear programming problem.

In period 2 the supplier's revenue-maximizing price offer depends on his (potentially) updated belief $\tilde{\beta}$ about the consumer's type. The supplier will post a high price $p = \theta^H$, accepted only by the high type, when his belief is $\tilde{\beta} > \frac{\theta^L}{\theta^H}$. When $\tilde{\beta} \leq \frac{\theta^L}{\theta^H}$, the supplier will post a low price $p = \theta^L$, accepted by both types.[13] The supplier's *future gains from trade* are thus given by

$$\tilde{V}(\tilde{\beta}) = \begin{cases} \tilde{\beta}\theta^H & \text{if } \tilde{\beta} > \frac{\theta^L}{\theta^H} \\ \theta^L & \text{if } \tilde{\beta} \leq \frac{\theta^L}{\theta^H}. \end{cases} \tag{7}$$

The consumer's future gains from trade are $\tilde{U}^L(\tilde{\beta}) = 0$ for the low type and

$$\tilde{U}^H(\tilde{\beta}) = \begin{cases} 0 & \text{if } \tilde{\beta} > \frac{\theta^L}{\theta^H} \\ \Delta\theta & \text{if } \tilde{\beta} \leq \frac{\theta^L}{\theta^H} \end{cases} \tag{8}$$

for the high type.

The supplier's updated belief $\tilde{\beta}$ depends on the consumer's contract choice in period 1. Given that the null-contract $(d, r, t) = (0, 0, 0)$ can always be included in the supplier's

[13]Although for $\tilde{\beta} = \frac{\theta^L}{\theta^H}$ the supplier is indifferent between the prices θ^L and θ^H in period 2, from a period 1 perspective pooling is preferable, as it reduces the high type's reluctance to reveal his type.

menu, it is without loss of generality to restrict attention to equilibria in which at least one contract becomes accepted. In equilibrium, the consumer's choice and the corresponding updated belief of the supplier can therefore take at most two values, $\tilde{\beta}_L$ and $\tilde{\beta}_H$, depending on whether the L- or the H-contract was accepted. Off equilibrium, that is following a rejection of all contracts, we can freely choose $\tilde{\beta} = 1$, implying a zero continuation payoff for the consumer, thereby maximizing the supplier's attainable payoff. We abbreviate notation by defining $\tilde{V}_m = \tilde{V}(\tilde{\beta}_m)$ and $\tilde{U}^i_m = \tilde{U}^i(\tilde{\beta}_m)$ and by letting $Q_H = \beta q^H + (1-\beta)q^L$ and $Q_L = 1 - Q_H$ denote the ex-ante probability that contract H or L are selected, respectively. Then Bayesian updating implies that the supplier's posterior beliefs about the consumer's type are given by

$$\tilde{\beta}_L \equiv \frac{\beta(1-q^H)}{Q_L} \quad \text{and} \quad \tilde{\beta}_H \equiv \frac{\beta q^H}{Q_H} \tag{9}$$

and it follows from $q^H \geq q^L$ that $\tilde{\beta}_L \leq \beta \leq \tilde{\beta}_H$.

In period 1, the supplier's problem can be formulated as the choice of a menu of contracts $(d_L, r_L, t_L), (d_H, r_H, t_H) \in [0,1]^2 \times \Re_+$ and the recommendation of a contract-choice strategy, $q^L \in [0,1]$ and $q^H \in [0,1]$, to the consumer that maximizes the supplier's expected revenue

$$V = \sum_{m \in \{L,H\}} Q_m[t_m + (1 - d_m + d_m r_m)\delta_S \phi \tilde{V}_m] \tag{10}$$

subject to the consumers' incentive and participation constraints

$$U^H_H \geq U^H_L \quad \text{with equality if} \quad q^H < 1 \tag*{(IC^H)}$$

$$U^L_L \geq U^L_H \quad \text{with equality if} \quad q^L > 0 \tag*{(IC^L)}$$

$$U^H_H \geq 0 \tag*{(PC^H)}$$

$$U^L_L \geq 0 \tag*{(PC^L)}$$

with U^i_m given by (1). Note that the incentive constraints have to hold with equality whenever the supplier recommends the consumer to randomize his choice of contract.

We show in the proof of Lemma 2 that (PC^H) is redundant and that at the optimum (IC^H) and (PC^L) must hold with equality.[14] Substitution of the transfers that make these constraints binding

$$t_L^{**} = d_L[1+(1-r_L)\delta_C]\theta^L \tag{11}$$

$$\begin{aligned} t_H^{**} &= d_H[1+(1-r_H)\delta_C]\theta^H - d_L[1+(1-r_L)\delta_C]\Delta\theta \\ &\quad -\delta_C\phi\{[1-d_L(1-r_L)]\tilde{U}_L^H - [1-d_H(1-r_H)]\tilde{U}_H^H\} \end{aligned} \tag{12}$$

then leads to the following *reduced program*:

$$\begin{aligned} \max_{d_L,r_L,d_H,r_H,q^L,q^H} & \sum_{i\in\{L,H\}} Q_i\{[d_i+d_i(1-r_i)\delta_C]\theta^i + [1-d_i(1-r_i)]\phi[\delta_C\tilde{U}_i^i+\delta_S\tilde{V}_i]\} \\ & - Q_H\{[d_L+d_L(1-r_L)\delta_C]\Delta\theta + [1-d_L(1-r_L)]\phi\delta_C\tilde{U}_L^H\} \end{aligned} \tag{13}$$

$$\begin{aligned} \text{subject to} \quad & d_H[1+(1-r_H)\delta_C] - d_L[1+(1-r_L)\delta_C] \\ & -\frac{\phi\delta_C}{\Delta\theta}\{[1-d_L(1-r_L)]\tilde{U}_L^H - [1-d_H(1-r_H)]\tilde{U}_H^H\} \geq 0 \end{aligned} \tag{DMC}$$

with equality if $q^L > 0$.

Lemma 2. *In a Perfect Bayesian equilibrium, the supplier's revenue is maximized by offering a menu of short-term contracts* $\{(d_L^{**}, r_L^{**}, t_L^{**}), (d_H^{**}, r_H^{**}, t_H^{**})\}$ *and recommending a contract choice strategy* (q^L, q^H) *to the consumer that solve (11), (12), and (13).*

The supplier's reduced program in (13) exhibits the familiar trade-off between maximization of surplus (the first line of the objective) and minimization of information-rent left to the high type (the second line of the objective). The (DMC) constraint is a dynamic version of the monotonicity constraint, requiring the contract designed for the high type to offer a higher expected discounted value, $d_i[1+(1-r_i)\delta_C$, to the consumer than the contract designed for the low type.

[14] While this result is standard in static models, in our dynamic setting standard arguments only allow for the conclusion that at least one of the two incentive constraints must be binding. In fact, in a renting framework with a tough supplier, (IC^L) can be binding an (IC^H) can be slack at the optimum, when the supplier is restricted to price-posting (Breig, 2020). It is the possibility of a randomized delivery that eliminates any slackness of (IC^H) at the optimum.

To understand the role that heterogeneous discounting plays in our model, it is instructive to rewrite the supplier's continuation value when his posterior turns out to be $\tilde{\beta}_L \leq \frac{\theta^L}{\theta^H}$ and the product has not been sold in period 1 as

$$\phi\delta_S\left\{\tilde{\beta}_L\theta^H + (1-\tilde{\beta}_L)\left[\hat{\theta}^L(\beta) + (1-\frac{\delta_C}{\delta_S})(\frac{\beta}{1-\beta} - \frac{\tilde{\beta}_L}{1-\tilde{\beta}_L})\Delta\theta\right]\right\}, \tag{14}$$

using the low type's *virtual valuation*, $\hat{\theta}^L(\beta) = \theta^L - \frac{\beta}{1-\beta}\Delta\theta$, familiar from auction theory. The fact that $\hat{\theta}^L$ is evaluated at the prior β reflects the supplier's time inconsistency and $\hat{\theta}^L$ is augmented by a positive term because, due to the consumer's impatience, the supplier does not need to compensate the high type in period 1 for the full loss in future information rents.

5 Restriction to a homogeneous mode of trade

In this section we characterize the supplier's optimal menu of short-term contracts when trade is restricted to consist of either selling or renting. For this purpose we solve the supplier's reduced program in (13) under the restriction that $r_l = r_h = 0$, or $r_l = r_h = 1$, respectively. To build intuition for our subsequent results, our analysis is divided into two steps. In Step 1 we determine the optimal pooling menu, maximizing the supplier's revenue amongst all menus that induce both types of consumer to accept the same contract. A comparison between pooling with a selling contract and pooling with a renting contract reveals an essential determinant of the supplier's mode of trade that exists even in the absence of informational asymmetries. In Step 2 we then determine the supplier's choice between pooling and separation when he is (exogeneously) restricted to either selling or renting. An important insight of this second step is that, under short term contracting, separation can be beneficial to the supplier, as it allows him to defer at least part of his compensation – in form of information rents – to the future. From an applied viewpoint, this section highlights the peculiarities of markets where products can only be rented, e.g. because they consist of a service, or sold, e.g. because they require costly consumer-specific installation such as solar panels.

Proposition 2 (Optimal Pooling Menu)**.** *Under short-term contracting, the supplier's revenue-maximizing way of pooling both types of consumer is to offer the selling contract* $\{(d,r,t)\} = \{(1,0,(1+\delta_C)\theta^L)\}$ *for* $\phi \leq \frac{\delta_C}{\delta_S}$ *and the renting contract* $\{(d,r,t)\} = \{(1,0,\theta^L)\}$ *for* $\phi \geq \frac{\delta_C}{\delta_S}$*. Whereas the former implements a sale in period 1, the latter induces a rental in period 1 followed by another rental, conditional on trade being unobstructed in period 2.*

If the supplier decides to pool both types in period 1 then his choice of short-term contract is characterized by a simple trade-off. On the one hand, renting jeopardizes future revenues because trade may happen to be obstructed in period 2. On the other hand, selling requires an excessively discounted price from the viewpoint of a patient supplier. Ideally, the supplier would contractually guarantee in period 1 the future availability of his product to the consumer but defer the consumer's payment until period 2. Under short-term contracting no contract can fulfill these two objectives and neither selling nor renting achieves the payoff the supplier could obtain with the help of a long-term contract. Pooling with a rental is preferable to pooling with a sale if and only if the deferred but uncertain payment, $\delta_S\phi\theta^L$, for the product's second period use is larger than the certain but undeferred payment, $\delta_C\theta^L$. In summary, the supplier's maximum payoff from pooling types under short-term contracting is therefore given by

$$V^{Pool} = [1 + \max\{\delta_C, \phi\delta_S\}]\theta^L. \tag{15}$$

Next we consider the supplier's choice between pooling and separation. Restricting the supplier's mode of trade to consist exclusively of selling or renting, allows us to highlight the benefits of separation as well as the costs of being unable to screen by mode of trade.

Proposition 3 (Optimal Selling and Renting Menus). *If the supplier is restricted to a single mode of trade his revenue-maximizing menu of short-term contracts can be characterized as follows:*

1. *If the supplier can sell but not rent, then the optimal menu is given by the separating menu* $\{(d_L, r_L, t_L), (d_H, r_H, t_H)\} = \{(0, 0, 0), (1, 0, (1 + \delta_C)\theta^H - \delta_C \phi \Delta\theta)\}$ *for* $\beta \in (\beta^S, \frac{\theta^L}{\theta^H})$ *and by the pooling menu* $\{(d, r, t)\} = \{(1, 0, (1 + \delta_C)\theta^L)\}$ *for* $\beta \in (0, \beta^S)$. *The separating menu induces full separation of types* ($q^H = 1, q^L = 0$) *by selling to the high type in period 1 while postponing a sale to the low type until period 2.*

2. *If the supplier can rent but not sell, then the optimal menu is given by the separating menu* $\{(d_L, r_L, t_L), (d_H, r_H, t_H)\} = \{(1-\phi\delta_C, 1, (1-\delta_C\phi)\theta^L), (1, 1, \theta^H-(1-\delta_C\phi)\Delta\theta-\delta_C\phi\Delta\theta)\}$ *for* $\beta \in (\beta^R, \frac{\theta^L}{\theta^H})$ *and by the pooling menu* $\{(d, r, t)\} = \{((1, 0, (1+\delta_C)\theta^L)\}$ *for* $\beta \in (0, \beta^R)$. *The separating menu induces full separation of types* ($q^H = 1, q^L = 0$), *by renting to the high type in period 1 while mixing between rental and exclusion for the low type.*

In period 2, price equals θ^L *under pooling, whereas under separation, price is* θ^L *or* θ^H, *contingent on contract choice in period 1. The thresholds* $\beta^S = \frac{(1+\delta_C-\phi\delta_S)\theta^L}{(1+\delta_C-\phi\delta_C)\theta^H-\phi(\delta_S-\delta_C)\theta^L} > \beta^R = \frac{\delta_C\theta^L}{\delta_S\theta^H-(\delta_S-\delta_C)\theta^L}$ *are positive and strictly smaller than* $\frac{\theta^L}{\theta^H}$.

Proposition 3 demonstrates that, under short-term contracting, separation can become optimal even when long-term contracting would lead to pooling. To understand this result, note that under separation, a high type consumer's period 1 payment is decreased by $\delta_C\phi\Delta\theta$ to compensate him for his loss in period 2 information rents. Due to his patience, the patient supplier attaches a greater value, $\delta_S\phi\Delta\theta$, to this reduction in the consumer's information rent. Separation thus provides a form of payment deferral that is not available under pooling.

While payment deferral in form of a reduction in information rents offers a benefit, separation also comes at a cost. Both under selling and renting, separation requires that the low type is excluded from trade in period 1 with positive probability. Exclusion is necessary because when the supplier is restricted to a single mode of trade, exclusion is the only way in which one of the two contracts can be made less attractive to the consumer *allocationally.* When the likelihood of facing a low type is sufficiently large, i.e. when β is sufficiently low, pooling therefore continues to be preferred by the supplier, just as under long-term contracting. As the following section shows, this conclusion must no longer be valid when the supplier is able to separate types by offering alternative modes of trade.

6 The optimal menu of short-term contracts

In this section, we determine the supplier's optimal menu of short-term contracts by solving the reduced program (13) without any restrictions on the mode of trade. In particular, we allow for both renting and selling contracts and show that screening by mode of trade constitutes a feature of the supplier's optimal menu.

An important element of the supplier's optimal menu follows directly from inspection of the reduced program. In the Appendix we prove the following:

Lemma 3. *The optimal menu of short-term contracts contains a contract that allocates the product to the consumer during period 1 by setting $d_H^{**} = 1$. If $\phi < \frac{\delta_C}{\delta_S}$, the optimal menu offers the product for sale by setting $r_H^{**} = 0$, thereby guaranteeing the product's efficient allocation in both periods.*

Lemma 3 is reminiscent of the "no distortion at the top" result from static screening (e.g. Mussa and Rosen, 1978), but with two important differences. First, $d_H^{**} = 1$ and $r_H^{**} = 0$ are not sufficient to induce the efficient allocation for the high type, because in our dynamic setting it might turn out to be optimal to allow the high type to choose the contract designed for the low type by recommending $q_H < 1$ (see Proposition 6 in Section 7). Second, Lemma 3 guarantees that the supplier offers the efficient allocation, i.e. a sale, only when his optimal pooling menu favors selling over renting. In other words, any inefficiency in the contract offered to the high type must be driven by an *inherent* preference for renting held by the supplier and can therefore be understood as a consequence of his inability to defer payments under short-term contracting.

Completing the characterization of the supplier's optimal menu of short-term contracts by solving the reduced program in (13) is a simple yet tedious application of linear programming whose details can be found in the Appendix. To report the solution we define the following thresholds:

$$\underline{\phi}(\beta) \equiv \frac{\delta_C(\theta^L - \beta\theta^H)}{\delta_S(\theta^L - \beta\theta^H) + \beta(\delta_S - \delta_C)\Delta\theta} \tag{16}$$

$$\bar{\phi}(\beta) \equiv \min\left\{\frac{\delta_S\theta^H - (\delta_S - \delta_C)\theta^L}{\delta_S\theta^H}, \frac{\delta_C\theta^L}{\delta_S\beta\theta^H}\right\} \tag{17}$$

$$\underline{\beta} \equiv \frac{\delta_C\theta^L}{\delta_S\theta^H - (\delta_S - \delta_C)\theta^L} \in (0, \frac{\theta^L}{\theta^H}). \tag{18}$$

$\underline{\phi}$ and $\bar{\phi}$ are decreasing and such that $0 < \underline{\phi}(\beta) < \frac{\delta_C}{\delta_S} < \bar{\phi}(\beta) < 1$.

Proposition 4 (Optimal Menu - Soft Supplier)**.** *The soft supplier's optimal menu of short-term contracts* $\{(d_L^{**}, r_L^{**}, t_L^{**}), (d_H^{**}, r_H^{**}, t_H^{**})\}$ *features screening by mode of trade if* $\beta \in (0, \frac{\theta^L}{\theta^H})$ *and* $\phi \in [\underline{\phi}, \bar{\phi})$*. Types fully separate in period 1 (*$q^H = 1$, $q^L = 0$*), with the low type renting (*$d_L^{**} = 1, r_L^{**} = 1$, $t_L^{**} = \theta^L$*) and the high type either buying (*$d_H^{**} = 1$, $r_H^{**} = 0$, $t_H^{**} = (1+\delta_C)\theta^H - (1+\delta_C\phi)\Delta\theta$ *for* $\phi \leq \frac{\delta_C}{\delta_S}$*) or mixing between buying and renting (*$d_H^{**} = 1$, $r_H^{**} = 1 - \phi$, $t_H^{**} = (1 + \delta_C\phi)\theta^L$ *for* $\phi > \frac{\delta_C}{\delta_S}$*). For the remaining parameters the optimal menu can be characterized as follows:*

- *If* $\beta \in (0, \frac{\theta^L}{\theta^H})$ *and* $\phi \in (0, \underline{\phi}]$*, the optimal menu pools the consumer by selling to both types in period 1, i.e.* $d_i^{**} = 1$, $r_i^{**} = 0$*, and* $t_i^{**} = (1 + \delta_C)\theta^L$ *for* $i \in \{L, H\}$*.*

- *If* $\beta \in (\underline{\beta}, \frac{\theta^L}{\theta^H})$ *and* $\phi \in [\bar{\phi}, 1)$*, the optimal menu induces full separation of types (*$q^H = 1$, $q^L = 0$*) in period 1. It rents to the high type (*$d_H^{**} = 1$, $r_H^{**} = 1$, $t_H^{**} = \theta^L$*), and mixes between renting and exclusion for the low type, (*$d_L^{**} = 1 - \phi\delta_C$, $r_L^{**} = 1$, $t_L^{**} = (1 - \phi\delta_C)\theta^L$*).*

- *If $\beta \in (0, \underline{\beta})$ and $\phi \in [\bar{\phi}, 1)$, the optimal menu pools the consumer by renting to both types in period 1, i.e.* $d_i^{**} = 1$, $r_i^{**} = 1$, *and* $t_i^{**} = \theta^L$ *for* $i \in \{L, H\}$.

In period 2, price equals θ^L under pooling, whereas under separation, price is θ^L or θ^H, contingent on contract choice in period 1.

Figure 1 provides a graphical representation of Proposition 4. In the shaded areas,

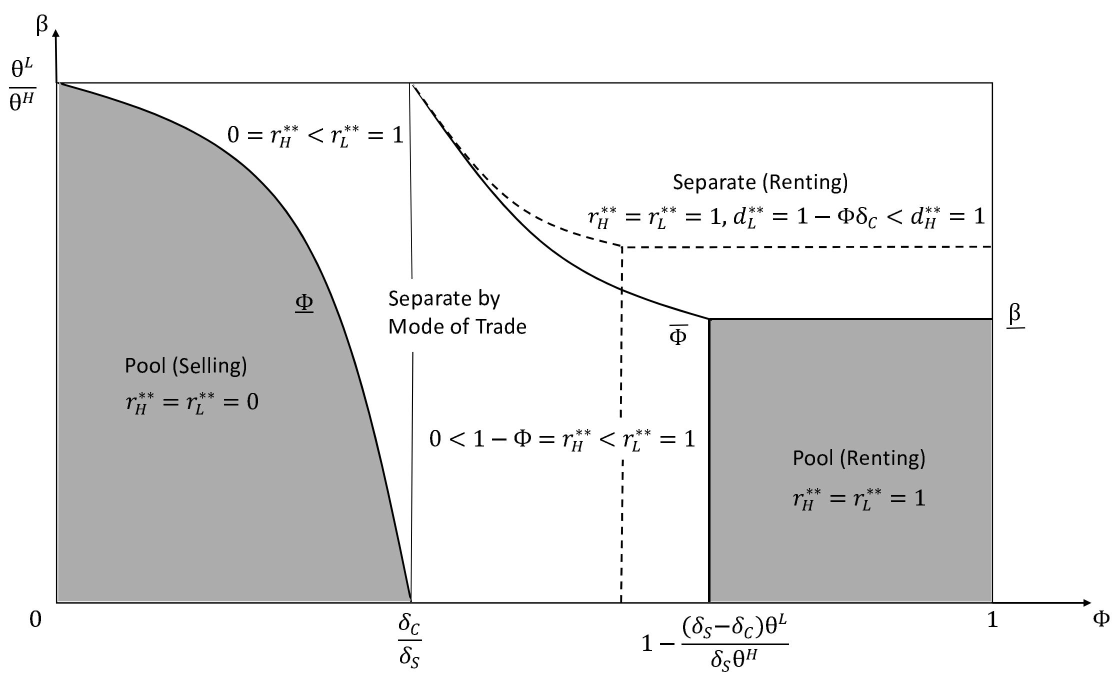

Figure 1: **Optimal Short-Term Contracting - Soft Supplier.** The supplier's revenue-maximizing menu of short-term contracts in dependence of his prior $\beta \in (0, \frac{\theta^L}{\theta^H})$ and the likelihood $\phi \in (0, 1)$ that trade in period 2 is unobstructed. Unless noted otherwise, the optimal menu sets not only $d_H^{**} = 1$ but also $d_L^{**} = 1$. The thresholds $\underline{\phi}$, $\bar{\phi}$, and $\underline{\beta}$ are as defined in (16), (17), and (18), respectively. The dashed lines depict the change in the thresholds $\bar{\phi}$ and $\underline{\beta}$ when the supplier is restricted to price-posting, leading to an expansion of the Pool (Renting) area.

the optimal menu of short-term contracts pools both types of the consumer by offering only one contract; a selling contract when the likelihood that future trade is unobstructed is low ($\phi < \underline{\phi}$), and a renting contract when this likelihood is high ($\phi > \bar{\phi}$).[15] In the

[15] In the limit, where $\phi \to 1$ and $\delta_S \to \delta_C$, Proposition 4 predicts the optimal menu to pool either via renting or via selling depending on the order in which these limits are taken. This is no contradiction, as

unshaded areas, the optimal menu induces full separation of consumer types. Note that the optimal menu induces separation in two mutually exclusive ways: (1) by decreasing the low type's probability of delivery d_L^{**} below d_H^{**}; or (2) by increasing the low type's probability of return r_L^{**} above r_H^{**}. Both, decreasing d_L or increasing r_L are possible means to achieve the monotonicity in "trade",

$$d_H[1+(1-r_H)\delta_C] - d_L[1+(1-r_L)\delta_C] \geq \delta_C[1-d_L(1-r_L)]\phi, \tag{19}$$

necessary for separation.[16] The possibility to reduce "trade" with the low type by renting rather than selling is a novel feature of short-term contracting in the presence of trade frictions, which becomes overlooked when the mode of trade is treated as exogenous.

Proposition 4 identifies screening by mode of trade as a possible characteristic of monopolistic short-term contracting and shows that, for a patient supplier, separation of types can be optimal even for arbitrarily low priors. It is important to note that, in our setting, the optimal menu of short-term contracts differs from the optimal menu of long-term contracts. In particular, under short-term contracting the soft supplier's payoff is strictly smaller than under long-term contracting, and the optimal menu may induce separation rather than pooling. This distinguishes our setting from Hart and Tirole (1988) where, for a soft supplier, the lack of commitment has no consequences. Our theory thus identifies novel effects of a monopolist's lack of commitment which arise when the assumptions of homogeneous discounting and the absence of trade-frictions are relaxed.

To understand why screening by mode of trade can be optimal, note that for $\phi \geq \frac{\delta_C}{\delta_S}$, the supplier's payoff from screening by mode of trade can be written as

$$V^{**} = V^{Pool} + \beta(\delta_S - \delta_C)\phi\Delta\theta - \beta\phi(\phi\delta_S - \delta_C)\theta^H. \tag{20}$$

this limit coincides with Hart and Tirole (1988) where a soft supplier's payoff is maximized by pooling and equals $(1+\delta^C)\theta^L$, no matter whether pooling is induced via a repeated rental or a sale. For $\phi \to 1$ and $\delta_S > \delta_C$ renting is optimal because it allows for payment deferral with no risk of trade frictions. For $\phi < 1$ and $\delta_S \to \delta_C$ selling is optimal because it guarantees the product's efficient allocation in period 2 without any financial loss from an advanced payment in period 1.

[16] For a separating menu, the consumer's second period gains from trade are $\tilde{U}_L^H = \Delta\theta$ and $\tilde{U}_H^H = 0$ and (19) follows from substitution into (DMC).

It exceeds the payoffs from pooling by a term that reflects that screening by mode of trade allows a patient supplier to defer part of his compensation to the future in the form of a reduction in the high type's information rents. Its cost derives from the fact that screening by mode of trade entails the possibility of a sale (to the high type with probability ϕ), although for $\phi > \frac{\delta_C}{\delta_S}$, renting constitutes the supplier's preferred mode of trade. For $\phi < 1 - \frac{(\delta_S - \delta_C)\theta^L}{\delta_S \theta^H}$ the benefit from a payment deferral via separation outweighs the cost of implementing the "wrong" mode of trade, and, because both benefit and cost are incurred with the same type of consumer, separation becomes optimal independently of the monopolist's prior. This distinguishes screening by mode of trade from ordinary screening (with one mode of trade) where the cost is incurred with the low type in the form of a reduction in first period trade, and screening becomes suboptimal for low priors.

An important implication of our theory is that, in the presence of trade frictions, informational asymmetries may be less persistent in a durable goods framework than commonly expected. Screening by mode of trade can occur even for those low values of the monopolist's prior for which ordinary screening would be prohibitively costly. Although it is not surprising that the supplier's patience improves his ability to screen, Proposition 4 demonstrates that the real power of heterogeneous discounting is unleashed when the mode of trade can be employed as a screening device.

Finally, a comment is in order regarding the use of randomization. The optimal menu may contain a contract that randomizes with respect to the product's delivery ($d_L^{**} = 1 - \phi\delta_C$) or with respect to the product's return ($r_H^{**} = 1 - \phi$). Interestingly, randomization is employed only for those parameters where renting constitutes the supplier's preferred mode of trade ($\phi \geq \frac{\delta_C}{\delta_S}$). This result resonates well with the existing literature which has found that randomization can be optimal in a renting framework (Beccuti and Möller, 2018) but not in a selling framework (Skreta, 2006).[17]

[17] Randomization can be optimal for a monopolist selling *multiple* varieties. See for example Thanassoulis (2004).

Randomization also induces the possibility that more general trading mechanisms with noisy communication (Bester and Strausz, 2007; Doval and Skreta, 2019a) become relevant. To see this, suppose that the monopolist observes a signal based on the consumer's choice of contract, or *message*, but not the consumer's message itself. If the mechanism rents to the low type but randomizes between selling and renting to the high type and renting is the outcome, then with noisy communication the monopolist must not necessarily learn the consumer's type. While Doval and Skreta (2019b) have shown that mechanisms with noisy communication cannot improve a monopolist's revenue in a selling framework, the emergence of randomization in connection with renting suggests that mechanisms with noisy communication might be optimal in a rental setting. We leave this issue for future research.

6.1 Price-posting

As much of the early literature confines attention to simple price posting (e.g. Stokey, 1981; Bulow, 1982; Gul et al., 1986; Ausubel and Deneckere, 1989), in the second part of this section, we reconsider the optimality of screening by mode of trade under a restriction to deterministic contracts. In practice, stochastic contracts are more difficult to verify and hence to enforce, and in some markets randomization might simply not be feasible. It is reassuring to see that, even in this restricted framework, screening by mode of trade emerges as part of the supplier's optimal trading strategy, although in a smaller subset of the parameter space.

In the following, we solve the supplier's contracting problem under the additional constraint that allocation and re-allocation probabilities must be either one or zero, i.e. $d_m, r_m \in \{0, 1\}$. This constraint is equivalent to requiring the supplier to set a rental and/or a sale price for his product in each period. Because Propositions 4 has shown that the supplier's optimal menu is deterministic if the likelihood of trade being unobstructed is low, we can restrict attention to the remaining case where $\phi > \frac{\delta_C}{\delta_S}$. The following definitions are necessary to state our result:

$$\underline{\beta}^{PP}(\phi) \equiv \frac{\theta^L}{\theta^H + \phi(\delta_S - \delta_C)\Delta\theta} \tag{21}$$

$$\bar{\phi}^{PP}(\beta) \equiv \min\left(\frac{\delta_C \theta^H}{\delta_C \Delta\theta + \delta_S \theta^L}, \frac{\theta^L - \beta\theta^H(1-\delta_C)}{\beta\delta_S\theta^H}\right). \tag{22}$$

Note that $\bar{\phi}^{PP} \in (\frac{\delta_C}{\delta_S}, \bar{\phi})$ and $\underline{\beta}^{PP} \in (\underline{\beta}, \frac{\theta^L}{\theta^H})$.

Proposition 5 (Price-Posting)**.** *A restriction to deterministic contracts makes separation less prevalent but screening by mode of trade continues to occur, even for arbitrarily low priors. In particular, the optimal price-posting menu is characterized as follows:*

- *If $\beta < \frac{\theta^L}{\theta^H}$ and $\phi \in (\frac{\delta_C}{\delta_S}, \bar{\phi}^{PP}]$, types are separated in period 1 by renting at price θ^L, accepted by the low type, and by selling at price $(1+\delta_C)\theta^H - (1+\delta_C\phi)\Delta\theta$, accepted by the high type.*

- *If $\phi \in [\bar{\phi}^{PP}, 1)$ and $\beta \geq \underline{\beta}^{PP}$, types are separated in period 1 by renting at price $\theta^H - \delta_C\phi\Delta\theta$, accepted only by the high type.*

- *If $\phi \in [\bar{\phi}^{PP}, 1)$ and $\beta \leq \underline{\beta}^{PP}$, types are pooled in period 1 by renting at price θ^L, accepted by both types.*

In period 2, price equals θ^L under pooling, whereas under separation, price is θ^H or θ^L, depending on whether the supplier's price in period 1 was accepted or rejected.

The monopolist's optimal price-posting menu is illustrated in Figure 1. The only difference to the fully optimal menu is a left-shift of the threshold $\bar{\phi}$ and an upward-shift of the threshold $\underline{\beta}$. The area of separation under price-posting is thus a strict subset of the area of separation when randomization is feasible. This is intuitive, because a restriction to deterministic menus affects the supplier's payoff from separation but not his payoff from pooling, making separation less prevalent. In conclusion, our analysis in this section has shown that the simple choice between a sale-price and a rental-price constitutes a powerful screening-device, capable of substituting inter-temporal discrimination by intra-period discrimination.

7 Tough supplier

In this section, we complete our analysis of monopolistic short-term contracting by considering the case of a tough supplier, the leading case of Hart and Tirole (1988). We show that, for a tough supplier, offering a menu consisting of both renting and selling contracts can never be optimal. The supplier separates or semi-separates the consumer by *either* renting *or* selling, just as in Hart and Tirole (1988). Our discovery of screening by mode of trade for a soft supplier thus complements Hart and Tirole's finding of semi-separation for a tough supplier, as special features of contracting in the absence of commitment.

For the remainder, let the supplier's prior belief be such that $\beta \geq \frac{\theta^L}{\theta^H}$. Define

$$\bar{\beta}(\phi) \equiv \begin{cases} \left(1 + \frac{\phi\delta_C\Delta\theta^2}{\theta^L(\theta^H + \delta_C\theta^H - \phi\delta_S\theta^L)}\right)^{-1} & \text{if} \quad \phi \leq \frac{\delta_C}{\delta_S} \\ \left(1 + \frac{\phi\delta_C\Delta\theta^2}{\theta^L(\theta^H + \phi\delta_S\theta^H - \phi\delta_S\theta^L)}\right)^{-1} & \text{if} \quad \phi \geq \frac{\delta_C}{\delta_S} \end{cases} \tag{23}$$

and note that $\bar{\beta}$ is decreasing and such that $\bar{\beta}(\phi) \in (\frac{\theta^L}{\theta^H}, 1)$.

Proposition 6 (Optimal Menu - Tough Supplier)**.** *The tough supplier's optimal menu of short-term contracts* $\{(d_L^{**}, r_L^{**}, t_L^{**}), (d_H^{**}, r_H^{**}, t_H^{**})\}$ *is implementable via simple price-posting and never combines renting and selling. Its details are as follows:*

- $\phi \in (0, \frac{\delta_C}{\delta_S}]$ *and* $\beta \leq \bar{\beta}(\phi)$*: Separate types in period 1 by selling at price* $(1+\delta_C)\theta^H - \delta_C\phi\Delta\theta$ *accepted only by the high type.*

- $\phi \in (0, \frac{\delta_C}{\delta_S}]$ *and* $\beta \geq \bar{\beta}(\phi)$*: Semi-separate types in period 1 by selling at price* $(1 + \delta_C)\theta^H$ *accepted only by the high type with probability* $q^H = \frac{\beta\theta^H - \theta^L}{\beta\Delta\theta}$.

- $\phi \in [\frac{\delta_C}{\delta_S}, 1)$ *and* $\beta \leq \bar{\beta}(\phi)$*: Separate types in period 1 by renting at price* $\theta^H - \delta_C\phi\Delta\theta$ *accepted only by the high type.*

- $\phi \in [\frac{\delta_C}{\delta_S}, 1)$ *and* $\beta \geq \bar{\beta}(\phi)$*: Semi-separate types in period 1 by renting at price* θ^H *accepted only by the high type with probability* $q^H = \frac{\beta\theta^H - \theta^L}{\beta\Delta\theta}$.

In period 2, price equals θ^H *under semi-separation, whereas under separation, price is* θ^L *or* θ^H*, contingent on contract choice in period 1.*

A graphical representation of Proposition 6 can be seen in Figure 2. Just as under the

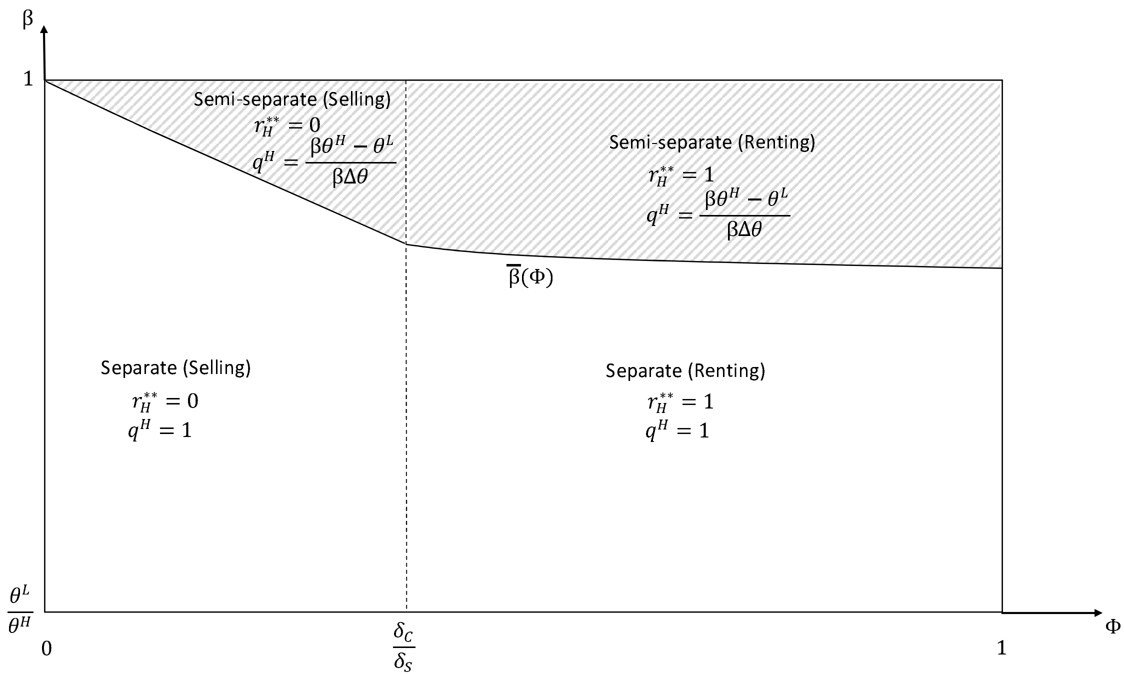

Figure 2: **Optimal Short-Term Contracting - Tough Supplier.** The supplier's revenue-maximizing menu of short-term contracts in dependence of his prior $\beta \in (\frac{\theta^L}{\theta^H}, 1)$ and the likelihood $\phi \in (0, 1)$ that trade in period 2 is unobstructed. In the depicted range of parameters, the optimal contracts set $d_H^{**} = 1$ and $d_L^{**} = 0$ (making r_L^{**} irrelevant), and induce rejection of the posted price by the low type, $q^L = 0$. The threshold $\bar{\beta}(\phi)$ is as defined in (23).

optimal pooling contract (see Proposition 2), the supplier sells when the likelihood that trade is unobstructed in period 2 is low ($\phi \leq \frac{\delta_C}{\delta_S}$) but rents when this likelihood is high ($\phi \geq \frac{\delta_C}{\delta_S}$). Concerning the degree of revealed information, Proposition 6 is reminiscent of Bolton and Dewatripont's (2005) textbook analysis of the case where $\phi = 1$ and $\delta_C = \delta_S$. In particular, the monopolist either separates or semi-separates types by inducing the high type to accept his price either with certainty or with probability $q^H = \frac{\beta\theta^H - \theta^L}{\beta\Delta\theta}$. Semi-separation allows the supplier to maintain posterior beliefs sufficiently high to charge the price θ^H in the future, thereby reducing the high type's information rent.

To understand why, for high priors, screening by mode of trade fails to be employed, remember that for $\beta \geq \frac{\theta^L}{\theta^H}$ the supplier would implement trade only with the high type in a static (one-period) context. Ordinary screening (with one mode of trade) achieves this objective by excluding the low type in period one. Moreover, ordinary screening implements the "right mode of trade" (selling for $\phi < \frac{\delta_C}{\delta_S}$, renting for $\phi > \frac{\delta_C}{\delta_S}$) with both types whereas screening by mode of trade offers the "wrong mode of trade" to one type. Ordinary screening is thus preferred over screening by mode of trade when the supplier is tough.

8 Continuum of types

Our previous analysis has shown that a monopolist's inability to offer multiple modes of trade can decrease his revenue when he is soft ($\beta < \frac{\theta^L}{\theta^H}$) but has no consequence when he is tough ($\beta \geq \frac{\theta^L}{\theta^H}$) . On the contrary, it is well established that a monopolist's inability to commit to future prices is harmful when he is tough but irrelevant when he is soft. The fact that one restriction matters when the other is irrelevant constitutes an artifact of the binary-type assumption. In this section, we extend our analysis to a setting with a continuum of types where both restrictions become relevant. We show that a monopolist's inability to offer multiple modes of trade can be more harmful than his inability to commit to future prices.

To simplify the subsequent analysis we assume that the consumer's type is uniformly distributed on the interval $[0, 1]$.[18] We restrict the supplier to simple price-posting. In period 1 the supplier offers a rental price $r \geq 0$ and a sale price $p \geq 0$. Restrictions on

[18]With a continuum of types, the mechanism design approach employed in this article is no longer valid. Skreta (2006) and Doval and Skreta (2019a) provide techniques to deal with a continuum of types.

the supplier's mode of trade can be captured by the requirement that $r = \infty$ (selling) or $p = \infty$ (renting). In period 2 the supplier can condition his price on whether or not the consumer has rented his product in period 1. Let q_r and q_n denote the prices offered to a returning and a new customer, respectively. With commitment, the monopolist can commit to q_r and q_n already in period 1, whereas without commitment, q_r and q_n must maximize the monopolist's second period revenue.[19] The consumer's expected payoff from buying, renting, or staying out of the market in period 1 are given by

$$U^{buy}(\theta) = \theta(1 + \delta_C) - p, \tag{24}$$

$$U^{rent}(\theta) = \theta - r + \phi\delta_C \max(\theta - q_r, 0), \tag{25}$$

$$U^{out}(\theta) = \phi\delta_C \max(\theta - q_n, 0). \tag{26}$$

The following skimming property is straightforward to show: If θ prefers buying over renting then so does $\theta' > \theta$. Similarly, if θ prefers renting over staying out then so does $\theta' > \theta$. Hence, given the supplier's pricing, the consumer's behavior in period 1 can be described by use of two thresholds $\underline{\theta}$ and $\bar{\theta}$ satisfying $0 \leq \underline{\theta} \leq \bar{\theta} \leq 1$: Types $\theta \in [\bar{\theta}, 1]$ buy, types $\theta \in [\underline{\theta}, \bar{\theta}]$ rent, and types $\theta \in [0, \underline{\theta}]$ stay out of the market in period 1.

When the supplier lacks commitment, prices in period 2 must be sequentially optimal which implies that

$$q_n^{**} \in \arg\max_{q_n} q_n(\underline{\theta} - q_n) \Rightarrow q_n^{**} = \frac{\underline{\theta}}{2} \tag{27}$$

$$q_r^{**} \in \arg\max_{q_r} q_r(\bar{\theta} - q_r) \Rightarrow q_r^{**} = \max\{\underline{\theta}, \frac{\bar{\theta}}{2}\}. \tag{28}$$

There are two possibilities. Either in period 2 the supplier repeats trade with all types that rented in period 1, i.e. $\underline{\theta} \geq \frac{\bar{\theta}}{2}$. Or, $\underline{\theta} < \frac{\bar{\theta}}{2}$, i.e. the supplier refrains from (repeating) trade with types in $[\underline{\theta}, \frac{\bar{\theta}}{2}]$ although he does trade (for the first time) with (lower) types in $[\frac{\underline{\theta}}{2}, \underline{\theta}]$. The first possibility turns out to be the relevant one. Hence, suppose that $\underline{\theta} \geq \frac{\bar{\theta}}{2}$

[19]Note that the notion of commitment employed in this section differs from Section 3 because long-term contracts, and hence the possibility of payment deferrals, are explicitly ruled out by our focus on price-posting.

(which has to be confirmed later). Then $q_r^{**} = \underline{\theta}$ and we can use the indifference conditions $U^{out}(\underline{\theta}) = U^{rent}(\underline{\theta})$ and $U^{rent}(\bar{\theta}) = U^{buy}(\bar{\theta})$ to express first period prices as a function of the indifferent types $\underline{\theta}$ and $\bar{\theta}$:

$$\begin{aligned} p &= \bar{\theta}\delta_C(1-\phi) + \underline{\theta}(1 + \frac{\phi\delta_C}{2}) && (29)\\ r &= \underline{\theta}(1 - \frac{\phi\delta_C}{2}). && (30)\end{aligned}$$

Substitution of these prices together with $q_n^{**} = \frac{\underline{\theta}}{2}$ and $q_r^{**} = \underline{\theta}$ into the supplier's revenue gives revenue in dependence of the induced thresholds $\underline{\theta}$ and $\bar{\theta}$:

$$V(\underline{\theta}, \bar{\theta}) = S(\underline{\theta}, \bar{\theta}) - I(\underline{\theta}, \bar{\theta}) + \phi(\delta_S - \delta_C)[(\bar{\theta} - \underline{\theta})\underline{\theta} + (\underline{\theta} - \frac{\underline{\theta}}{2})\frac{\underline{\theta}}{2}]. \quad (31)$$

The supplier's payoff equals the difference between surplus S and information rents I given by

$$\begin{aligned} S(\underline{\theta}, \bar{\theta}) &= \int_{\frac{\underline{\theta}}{2}}^{\underline{\theta}} \phi\delta_C\theta d\theta + \int_{\underline{\theta}}^{\bar{\theta}} (1+\phi\delta_C)\theta d\theta + \int_{\bar{\theta}}^{1} (1+\delta_C)\theta d\theta && (32)\\ I(\underline{\theta}, \bar{\theta}) &= \int_{\frac{\underline{\theta}}{2}}^{\underline{\theta}} \phi\delta_C(\theta - \frac{\underline{\theta}}{2})d\theta + \int_{\underline{\theta}}^{\bar{\theta}} \phi\delta_C\frac{\underline{\theta}}{2} + (\theta - \underline{\theta})(1+\phi\delta_C)d\theta && (33)\\ &\quad + \int_{\bar{\theta}}^{1} \phi\delta_C\frac{\underline{\theta}}{2} + (\bar{\theta} - \underline{\theta})(1+\phi\delta_C) + (\theta - \bar{\theta})(1+\delta_C)d\theta, \end{aligned}$$

plus a term arising from the deferral of trade to period 2 via renting. In the Appendix we prove the following:

Proposition 7. *Suppose that $\theta \in [0,1]$ is uniformly distributed. If the supplier is sufficiently patient, i.e. $\delta_S > 2\delta_C$, and prefers selling over renting, i.e. $\phi < \frac{\delta_C}{\delta_S}$, then his revenue is maximized by inducing the thresholds*

$$\begin{aligned} \bar{\theta}^{**} &= \frac{\delta_C(1-\phi)[4+\phi\delta_S+2\phi(\delta_S-\delta_C)] + \phi(\delta_S-\delta_C)(2+\phi\delta_C)}{2\delta_C(1-\phi)[4+\phi\delta_S+2\phi(\delta_S-\delta_C)] - 2\phi^2(\delta_S-\delta_C)^2} && (34)\\ \underline{\theta}^{**} &= \frac{2+\phi\delta_C+2\phi(\delta_S-\delta_C)\bar{\theta}^{**}}{4+\phi\delta_C+2\phi(\delta_S-\delta_C)}, && (35)\end{aligned}$$

and $0 < \underline{\theta}^{**} < \bar{\theta}^{**} < 1$, that is, screening by mode of trade is optimal.

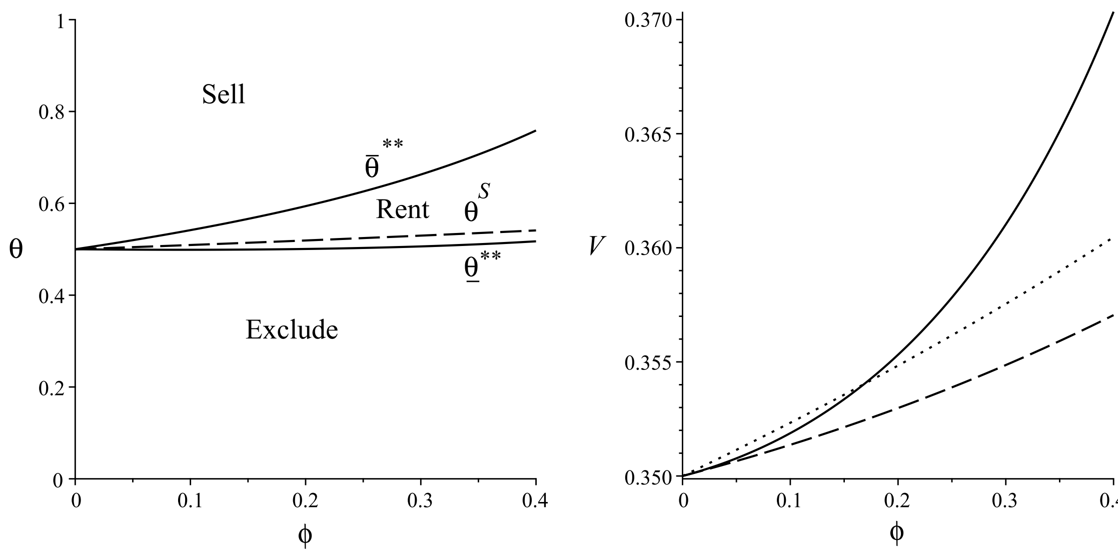

Figure 3: **Mode of Trade and Revenue for Uniform Type Distribution.** The left hand panel shows the segments of types, $[\underline{\theta}^{**}, \bar{\theta}^{**}]$ and $[\bar{\theta}^{**}, 1]$, that are induced to rent or buy under screening by mode of trade in dependence of the likelihood of a future trade opportunity ϕ. Comparison is with the threshold θ^S the monopolist would induce if selling was his only mode of trade. The right hand panel depicts revenue under screening by mode of trade (solid) compared with revenue under selling, with commitment (dotted) and without commitment (dashed) to future prices. For large enough ϕ a monopolistic seller loses more from the non-availability of a rental-option than from his inability to commit to future prices. Parameter values are $\delta_S = 1$ and $\delta_C = 0.4$.

The thresholds $\underline{\theta}^{**}$ and $\bar{\theta}^{**}$ are depicted in Figure 3. Comparison with the threshold

$$\theta^S = \frac{2 + (2-\phi)\delta_C}{4 + 2(2-\phi)\delta_C - \phi\delta_S} \tag{36}$$

that the supplier would induce if he was restricted to selling reveals that $\underline{\theta} < \theta^S < \bar{\theta}$.[20] Hence, under screening by mode of trade, the supplier trades with some types that under selling would be excluded, while inducing other types to only rent rather than buy.

The continuous type model can be used to understand why screening by mode of trade remains optimal even when renting becomes arbitrarily unattractive, i.e. for ϕ arbitrarily small. For this purpose, consider a move from pure selling with cutoff θ^S to screening by mode of trade with a renting-interval $[\underline{\theta}, \bar{\theta}] = [\theta^S - \frac{\epsilon}{2}, \theta^S + \frac{\epsilon}{2}]$. In the limit where $\phi \to 0$,

[20] The supplier's revenue maximizing pricing strategy in the pure selling case is derived in the Appendix.

the corresponding changes in surplus and information rent are given by

$$\lim_{\phi\to 0}\Delta S = \frac{1}{2}\theta^S(1-\delta_C)\epsilon + O(\epsilon^2) \quad (37)$$

$$\lim_{\phi\to 0}\Delta I = -\frac{1}{2}\theta^S(1-\delta_C)\epsilon - \frac{1}{2}\delta_C\epsilon + O(\epsilon^2). \quad (38)$$

For small ϵ, screening by mode of trade raises surplus because the surplus gain of types $\theta \in [\tilde{\theta}-\frac{\epsilon}{2}, \tilde{\theta}]$ who move from exclusion to renting, $\lim_{\phi\to 0}[(1+\phi\delta_C)-\phi\delta_C]\theta = \theta$, exceeds the surplus loss of types $\theta \in [\tilde{\theta}, \tilde{\theta}+\frac{\epsilon}{2}]$ who move from buying to renting, $\lim_{\phi\to 0}[(1+\phi\delta_C)-(1+\delta_C)]\theta = -\delta_C\theta$. In addition, screening by mode of trade allows the supplier to reduce the consumer's information rent because it allows him to differentiate not only between non-traders and buyers but between non-traders, renters, and buyers. In summary, screening by mode of trade is optimal even when renting is arbitrarily unattractive, because the surplus loss of renting arises only in the future, whereas the surplus loss of excluding the consumer from trade arises already in period 1. Although our analysis has been restricted to a uniform distribution of types, we expect the above argument to apply to more general type distributions.

Figure 3 also shows the supplier's revenue under screening by mode of trade in comparison to the case of pure selling with commitment (dotted) and without commitment (dashed) to future prices. As one can see from the figure, the monopolist's revenue becomes reduced both by his lack of commitment and by a restriction to a single mode of trade. More importantly, Figure 3 reveals that a monopolistic seller may lose less from his lack of commitment to future prices than from a restriction to a single mode of trade. This finding underlines the relevance of endogenizing a monopolist's choice between selling and renting in a durable goods setting with an uncertain trade-horizon.

9 Conclusion

In this article, we have determined the revenue-maximizing menu of short-term contracts for a monopolistic, non-anonymous durable goods market subject to trade frictions. While we have put minimal restrictions on the set of feasible contracts, the analysis was simplified by our focus on a two-period framework. Before we summarize our main findings, we provide a brief discussion of the potential effects of allowing for longer horizons.

Extending our two-type model to allow for three periods of trade, we have been able to show that, under price-posting, screening by mode of trade continues to be optimal for a soft supplier.[21] Although the area of the parameter space where screening by mode of trade is optimal is smaller than in the two-period case, we suspect that even for arbitrary horizons, screening by mode of trade will emerge. The reason is that, when the presence of trade frictions and the inability to offer long-term contracts makes the patient supplier indifferent between selling and renting, screening by mode of trade comes at zero cost while providing an indirect form of payment-deferral. Interestingly, with three periods, screening by mode of trade can be optimal also in the case of a tough supplier, which has been the focus of the existing literature. This indicates that existing results will remain valid when discount factors are assumed to be homogeneous or trade frictions are assumed to be absent but modifications can be expected when both assumptions are dropped simultaneously.

In summary, the main message of this article is that in a durable goods market with trade-frictions and heterogeneous discounting, screening by mode of trade will emerge as a feature of a monopolist's optimal menu of short-term contracts. Screening by mode of trade allows a monopolist to defer part of his compensation to the future, in form of a reduction in consumers' information rents, while minimizing (potentially to zero) the allocational cost of separation from the viewpoint of a patient supplier.

[21] With more than two periods, optimal short-term contracting may involve mechanisms more sophisticated than the ones considered in this article. In particular, it might be optimal from the supplier's perspective in period 1 to offer more than two contracts in period 2, in order to fine-tune beliefs in period 3.

Appendix: Proofs

Proof of Lemma 1. We show that the supplier's mechanism design problem in period 1 is isomorphic to a static mechanism design problem a la Bester and Strausz (2001). Lemma 1 then follows directly from Proposition 2 of Bester and Strausz (2001). For this purpose, using the language of Bester and Strausz (2001), define the sets X and Y of the supplier's contractible decisions and non-contractible decisions, respectively, as follows:

$$X = \{(d, r, t) | d, r \in [0, 1], t \in \Re_+\} \tag{39}$$

$$Y = \{p | p \in \{\theta^L, \theta^H\}\}. \tag{40}$$

Note that here we have made use of the fact that in a Perfect Bayesian equilibrium all price offers $p \notin \{\theta^L, \theta^H\}$ can never be optimal, independently of the supplier's updated belief. Given the supplier's decisions (d, r, t) and p and the consumer's type $i \in \{L, H\}$, the supplier's and the consumer's payoffs are given by:

$$V^i((d, r, t), p) = t + \phi(1 - d + dr)\delta_S R_i(p) \tag{41}$$

$$U^i((d, r, t), p) = \theta^i d[1 + (1 - r)\delta_C] - t + \phi(1 - d + dr)\delta_C \max(0, \theta^i - p). \tag{42}$$

Both X and Y are metric spaces (Y can be endowed with the discrete metric) and given that Y is discrete, both V^i and U^i are continuous functions on $X \times Y$. Hence, the conditions of Bester and Strausz (2001) are satisfied and their Proposition 2 implies our Lemma 1. □

Proof of Proposition 1. Following standard arguments from static mechanism design, the participation constraint (4) is redundant for $i = H$ and must be binding for $i = L$. Similarly, the incentive constraint (5) is redundant for $i = L$ and must be binding for $i = H$. Hence deferred transfers in the optimal menu of long-term contracts must satisfy

$$t_L^2 = \frac{1}{\delta_C \phi} x_L \theta^L \tag{43}$$

$$t_H^2 = \frac{1}{\delta_C \phi}(x_H \theta^H - x_L \Delta\theta). \tag{44}$$

Substitution into the supplier's objective (3) gives the unconstrained program

$$\max_{x_L, x_H} \frac{\delta_S}{\delta_C}[\beta(x_H\theta^H - x_L\Delta\theta) + (1-\beta)x_L\theta^L]. \tag{45}$$

The objective is increasing in x_H. Because $\beta < \frac{\theta^L}{\theta^H}$ the objective is also increasing in x_L. Hence the optimal menu must let x_L and x_H take their highest possible values, i.e. $x_L = x_H = 1 + \delta_C$, which requires $d_H^1 = d_L^1 = 1$ and $r_H^1 = r_L^1 = 0$. The optimal long-term contract thus sells to both types in period 1 and the consumer makes the payment $t_L^2 = t_H^2 = \frac{(1+\delta_C)\theta^L}{\delta_C\phi}$ in period 2 if trade happens to be unobstructed. □

Proof of Lemma 2. In a Perfect Bayesian equilibrium, the supplier maximizes his revenue in (10) subject to the incentive constraints, (IC^H), (IC^L), and the participation constraints, (PC^H), (PC^L). As $\theta^H > \theta^L$ and $\tilde{U}_L^H \geq \tilde{U}_L^L = 0$, (IC^H) and (PC^L) together imply (PC^H). Hence (PC^H) is redundant. If (PC^L) holds with strict inequality then raising both transfers t_L and t_H by a sufficiently small and identical amount increases the supplier's objective while keeping all constraints satisfied. Hence the optimal menu of short-term contracts must make (PC^L) binding. Similarly, if both (IC^L) and (IC^H) hold with strict inequality then raising t_H by a sufficiently small amount increases the objective while maintaining both inequalities strict. Hence, at least one incentive constraint must hold with equality. In order to derive the reduced program, we assume that (IC^H) is binding and then substitute the payments (11) and (12), that make (PC^L) and (IC^H) hold with equality, into the remaining constraint (IC^L) and the objective (10) to obtain (DMC) and (13), respectively. To see that this final step is without loss of generality, assume alternatively, that (IC^L) is binding. Substitution of the payments that make (PC^L) and (IC^L) hold with equality,

$$t_i = d_i[1 + (1 - r_i)\delta_C]\theta^i, \quad i \in \{L, H\} \tag{46}$$

into the remaining constraint (IC^H) and the objective (10) leads to the following program:

$$\max_{d_L,r_L,d_H,r_H,q^L,q^H} \sum_{i\in\{L,H\}} Q_i\{[d_i + d_i(1-r_i)\delta_C]\theta^i + [1-d_i(1-r_i)]\phi[\delta_S\tilde{V}_i + \delta_C\tilde{U}_i^i]\} \tag{47}$$
$$- Q_H\{[d_L + d_L(1-r_L)\delta_C]\Delta\theta + [1-d_L(1-r_L)]\phi\delta_C\tilde{U}_L^H\}$$
$$- Q_H\Delta\theta\{d_H[(1+(1-r_H)\delta_C] - d_L[(1+(1-r_L)\delta_C]\}$$
$$+ Q_H\phi\delta_C\{[1-d_L(1-r_L)]\tilde{U}_L^H - [1-d_H(1-r_H)]\tilde{U}_H^H\}$$

$$\text{subject to} \quad d_H[(1+(1-r_H)\delta_C] - d_L[(1+(1-r_L)\delta_C] \tag{DMC'}$$
$$- \frac{\phi\delta_C}{\Delta\theta}\{[1-d_L(1-r_L)]\tilde{U}_L^H - [1-d_H(1-r_H)]\tilde{U}_H^H\} \geq 0$$

with equality if $q^H < 1$.

Note that the objective (47) is identical to the objective (13) of the reduced program except for the last two lines and that the constraint (DMC') is the same as (DMC) except that it must hold with equality for $q^H < 1$ rather than for $q^L > 0$. Choosing $q^H < 1$ makes (DMC') binding and the last two lines of (47) become zero, i.e. (47) becomes identical to (13). Alternatively, setting $q^H = 1$ allows (DMC') to be slack, but this makes the last two lines of (47) negative. In other words, for any menu of contracts that solves the above program, we can find a menu that solves the reduced program and leads to (at least weakly) larger payoff. □

Proof of Proposition 2. If the supplier pools types by offering a single contract $\{(d, r, t)\}$ accepted by both types, then substitution of $d_L = d_H = d$, $r_L = r_H = r$, and $q^L = q^H$ into (13) and (DMC) simplifies the reduced program to the unconstrained program

$$\max_{d,r}[d + d(1-r)\delta_C]\theta^L + [1-d(1-r)]\phi\delta_S\theta^L = \max_{d,r} d\theta^L + d(1-r)(\delta_C - \phi\delta_S)\theta^L. \tag{48}$$

As $\delta_C - \phi\delta_S < 1$ the optimal pooling menu must set $d = 1$. Moreover, $r = 1$ (renting) is optimal when $\delta_C - \phi\delta_S \leq 0$, whereas $r = 0$ (selling) is optimal when $\delta_C - \phi\delta_S \geq 0$. The optimal transfer follows from the low type's binding participation constraint. □

Proof of Proposition 3. We consider the cases of selling and renting in turn and use the fact that, in both cases, setting $d_H = 1$ is optimal. We will prove this fact in the case where the mode of trade is unrestricted (Lemma 3). Analog arguments show that the insight remains valid when either selling or renting is ruled out.

Selling. Assume first that the supplier can sell but not rent, i.e. let $r_L = r_H = 0$. Substitution together with $d_H = 1$ simplifies the reduced program to the following *selling program*:

$$\max_{(d_L, q^L, q^H)} d_L[1+\delta_C]\theta^L + (1-d_L)\phi\delta_S\theta^L + Q_H\{[1+\delta_C]\theta^H - d_L[1+\delta_C]\theta^H - (1-d_L)\phi(\delta_C\Delta\theta + \delta_S\theta^L)\}$$

$$\text{s.t.} \quad d_L \leq 1 \quad \text{with equality if} \quad q^L > 0.$$

For $q^L > 0$ the constraint binds and the objective becomes equal to the pooling payoff $(1+\delta_C)\theta^L$. For $q^L = 0$ the program simplifies to the unconstrained program

$$\max_{(d_L, q^H)} d_L[1+\delta_C]\theta^L + (1-d_L)\phi\delta_S\theta^L + \beta q^H\{[1+\delta_C]\theta^H - d_L[1+\delta_C]\theta^H - (1-d_L)\phi(\delta_C\Delta\theta + \delta_S\theta^L)\}.$$

Linearity in d_L implies that $d_L \in \{0,1\}$. Setting $d_L = 1$ leads to the pooling payoff. For $d_L = 0$ the remaining objective is increasing in q^H and setting $q^H = 1$ gives the separating payoff $\phi\delta_S\theta_L + \beta[\theta_H + \delta_C\theta_H - \phi(\delta_S\theta_L + \delta_C\Delta\theta)]$. The seller prefers separation over pooling if and only if

$$\beta \geq \beta^S \equiv \frac{\theta^L(1+\delta_C - \phi\delta_S)}{\theta^H(1+\delta_C) - \phi(\delta_S\theta^L + \delta_C\Delta\theta)}.$$

Renting. Next assume that the supplier can rent but not sell, i.e. let $r_L = r_H = 1$. Substitution together with $d_H = 1$ simplifies the reduced program to the following *renting program*:

$$\max_{(d_L, q^L, q^H)} d_L\theta^L + \phi\delta_S\theta^L + Q_H\{\theta^H + \phi(\delta_C\tilde{U}_H^H + \delta_S\tilde{V}_H) - d_L\theta^H - \phi(\delta_C\Delta\theta + \delta_S\theta^L)\}$$

$$\text{s.t.} \quad 1 + \phi\delta_C\frac{\tilde{U}_H^H}{\Delta\theta} - \phi\delta_C \geq d_L \quad \text{with equality if } q^L > 0.$$

For any q^L, q^H inducing posterior $\tilde{\beta}_H \leq \theta^L/\theta^H$ it holds that $\tilde{U}_H^H = \Delta\theta$ and $\tilde{V}_H = \theta^L$ and the program's constraint is automatically satisfied. The fact that it has to hold with equality when $q^L > 0$ is equivalent to the requirement that $(1-d_L)q^L = 0$. Using this insight, it is easy to see that the remaining objective increases in d_L so that the supplier cannot do better than by setting $d_L = 1$, leading to the pooling payoff $\theta^L + \phi\delta_S\theta^L$. Alternatively, if the supplier chooses a q^L, q^H inducing posterior $\tilde{\beta}_H > \theta^L/\theta^H$ then $\tilde{U}_H^H = 0$ and $\tilde{V}_H = \tilde{\beta}_H\theta^H$. If $q_L > 0$, the program's constraint must bind, i.e. $d_L = 1 - \phi\delta_C$. If $q_L = 0$, then the program's objective is increasing in d_L because $\theta^L - Q_H\theta^H = \theta^L - \beta q^H\theta^H \geq \theta^L - \beta\theta^H > 0$ and it is thus optimal to choose the largest d_L that satisfies the constraint, i.e. again $d_L = 1 - \phi\delta_C$. In both cases, the supplier's payoff from separation becomes $\theta^L + (1-\beta)\phi(\delta_S - \delta_C)\theta^L + \beta\phi\delta_S\theta^H$. The supplier prefers separation over pooling if and only if

$$\beta \geq \frac{\delta_C\theta_L}{\delta_S\theta_H - (\delta_S - \delta_C)\theta_L} \equiv \beta^R.$$

□

Proof of Lemma 3. Taking the derivative of the supplier's objective in (13) with respect to d_H we obtain

$$Q_H\{\theta^H + (1-r_H)[\delta_C(\theta^H - \phi\tilde{U}_H^H) - \phi\delta_S\tilde{V}_H]\} > Q_H[\theta^H - (1-r_H)\phi\delta_S\tilde{V}_H] > 0, \quad (49)$$

where we have used the fact that $\tilde{U}_H^H \leq \Delta\theta < \theta^H$ and $\tilde{V}_H \leq \theta^H$. For the same reason, the derivative of the left hand side of the (DMC) constraint with respect to d_H is

$$1 + (1-r_H)\delta_C(1 - \frac{\tilde{U}_H^H}{\Delta\theta}) > 0. \quad (50)$$

Hence an increase in d_H increases the supplier's objective (13) while relaxing the constraint (DMC), so that setting $d_H^{**} = 1$ must be optimal.

Now consider the derivative of the supplier's objective in (13) with respect to r_H and note that for $\phi < \frac{\delta_C}{\delta_S}$ this derivative becomes

$$Q_H d_H\delta_C(\phi\tilde{U}_H^H + \frac{\phi\delta_S}{\delta_C}\tilde{V}_H - \theta^H) < Q_H d_H\delta_C(\tilde{U}_H^H + \tilde{V}_H - \theta^H) < 0. \quad (51)$$

Finally, the derivative of the left hand side of the (DMC) constraint with respect to r_H is

$$d_H \delta_C (\frac{\phi \tilde{U}_H^H}{\Delta\theta} - 1) < 0. \tag{52}$$

Hence for $\phi < \frac{\delta_C}{\delta_S}$, a reduction in r_H increases the supplier's objective while relaxing the (DMC) constraint, so that setting $r_H^{**} = 0$ must be optimal. □

Proof of Proposition 4. From $\tilde{\beta}_L \leq \beta < \frac{\theta^L}{\theta^H}$ we have $\tilde{U}_L^H = \Delta\theta$ and $\tilde{V}_L = \theta^L$. We make use of Lemma 3 by considering the cases $\phi < \frac{\delta_C}{\delta_S}$ and $\phi \geq \frac{\delta_C}{\delta_S}$ in separation.

Case $\phi < \frac{\delta_C}{\delta_S}$: Substitution of $d_H^{**} = 1$ and $r_H^{**} = 0$ into the reduced program gives

$$\begin{aligned} &\max_{(d_L, r_L, q^L, q^H)} d_L[1 + (1 - r_L)\delta_C]\theta^L + (1 - d_L + d_L r_L)\phi\delta_S\theta^L \\ &+Q_H\{(1 - d_l)\theta^H + (1 - d_l + d_l r_l)[(\delta_C - \phi\delta_S)\theta^H + \phi(\delta_S - \delta_C)\Delta\theta]\} \\ &\text{s.t.} \quad 1 + (1 - \phi)\delta_C - d_L[1 + (1 - r_L)(1 - \phi)\delta_C] \geq 0 \\ &\text{with equality if} \quad q^L > 0. \end{aligned} \tag{53}$$

If the supplier sets $(d_L^{**}, r_L^{**}) = (1, 0)$ then he pools by selling to both types and his payoff is given by

$$V^{SS} = (1 + \delta_C)\theta^L. \tag{54}$$

If the supplier chooses $(d_L, r_L) \neq (1, 0)$, then the constraint holds with strict inequality, and hence the low type must be induced to choose the L-contract, i.e. $q^L = 0$. Moreover, as the objective in (53) increases in Q_H, while neither the objective nor the constraint depend any longer on $\tilde{U}_H^H$ (because the menu sells to the high type), it must be optimal to induce the high type to choose the H-contract, i.e. $q^H = 1$. Substitution of $q^L = 0$ and $q^H = 1$ then leaves the following unconstrained program:

$$\begin{aligned} \max_{(d_L, r_L)} \; & d_L(\theta^L - \beta\theta^H) + (1 - r_L)d_L\{\delta_C(\theta^L - \beta\theta^H) - \phi[\delta_S(\theta^L - \beta\theta^H) + \beta(\delta_S - \delta_C)\Delta\theta]\} \\ & +\beta[(1 + \delta_C)\theta^H - \phi\delta_C\Delta\theta] + (1 - \beta)\phi\delta_S\theta^L. \end{aligned} \tag{55}$$

Consider the threshold $\underline{\phi}$ defined in (16) and note that $\beta < \frac{\theta^L}{\theta^H}$ and $\delta_C < \delta_S$ imply that $\underline{\phi} \in (0, \frac{\delta_C}{\delta_S})$. For $\phi \leq \underline{\phi}$ the objective in (55) is decreasing in r_L and increasing in d_L. Hence for $\phi \leq \underline{\phi}$ the optimal mechanism sets $(d_L^{**}, r_L^{**}) = (1, 0)$, i.e. it pools by selling to both types, and the supplier's maximized revenue is given by V^{SS}. For $\phi > \underline{\phi}$ the objective in (55) is increasing in r_L and, after substitution of $r_L^{**} = 1$, the remaining objective is clearly increasing in d_L. Hence, for $\phi \in (\underline{\phi}, \frac{\delta_C}{\delta_S})$ it is optimal to set $(d_L^{**}, r_L^{**}) = (1, 1)$, i.e. the optimal menu rents to the low type but sells to the high type. The corresponding payoff is given by

$$V^{RS} = \theta^L + \phi\delta_S\theta^L + \beta\{\delta_C\theta^H - \phi(\delta_C\Delta\theta + \delta_S\theta^L)\}. \tag{56}$$

This completes our characterization of the supplier's optimal mechanism for the case where $\phi < \frac{\delta_C}{\delta_S}$.

Case $\phi \geq \frac{\delta_C}{\delta_S}$*:* In this case an increase in r_L increases the objective (13) while relaxing the constraint (DMC). Substitution of $d_H^{**} = 1$ and $r_L^{**} = 1$ into the reduced program gives

$$\begin{aligned} \max_{(d_L, r_H, q^L, q^H)} \quad & d_L\theta^L + (1-Q_H)\phi\delta_S\theta^L + Q_H\{r_H\phi\delta_S\tilde{V}_H + [1+(1-r_H)\delta_C]\theta^H \\ + \quad & r_H\phi\delta_C\tilde{U}_H^H - d_L\theta^H - \phi\delta_C\Delta\theta\} \\ \text{s.t.} \quad & -d_L\Delta\theta - \phi\delta_C\Delta\theta + [1+(1-r_H)\delta_C]\Delta\theta + r_H\phi\delta_C\tilde{U}_H^H \geq 0 \\ & \text{with equality if} \quad q^L > 0. \end{aligned} \tag{57}$$

Accounting for the piecewise definition of $\tilde{V}_H$ and $\tilde{U}_H^H$, in the following we consider in separation two possible types of contract-menus: *Learning menus* that induce a posterior belief $\tilde{\beta}_H > \frac{\theta^L}{\theta^H}$; and *non-learning menus* which induce a posterior belief $\tilde{\beta}_H \leq \frac{\theta^L}{\theta^H}$.

Non-learning menus: If (q^L, q^H) are such that $\tilde{\beta}_H \leq \frac{\theta^L}{\theta^H}$, then $\tilde{U}_H^H = \Delta\theta$ and $\tilde{V}_H = \theta^L$, and the constraint in (57) can be written as $d_L \leq 1 + (1-r_H)(1-\phi)\delta_C$. A non-learning menu must set $r_H = 1$, because for $r_H < 1$ the constraint could not be binding, and the low type would be induced to choose contract L, resulting in $\tilde{\beta}_H = 1$. Setting $r_H^{**} = 1$ the constraint is automatically satisfied and the fact that it must hold with equality when $q^L > 0$ is equivalent to the requirement that $(1 - d_L)q^L = 0$. The problem simplifies to the unconstrained program

$$\max_{d_L, q^H} d_L\theta^L + \phi\delta_S\theta^L + \beta q^H(1 - d_L)\theta^H \tag{58}$$

whose objective is increasing in d_L. Setting $d_L^{**} = 1$, the corresponding contract-menu $\{(d_L^{**}, r_L^{**}), (d_H^{**}, r_H^{**})\} = \{(1, 1), (1, 1)\}$ pools by renting to both types, and leads the payoff

$$V^{RR} = (1 + \phi\delta_S)\theta^L. \tag{59}$$

We have thus shown that for $\phi \geq \frac{\delta_C}{\delta_S}$, V^{RR} is the highest payoff obtainable with a non-learning menu.

Learning menus: If (q^L, q^H) are such that $\tilde{\beta}_H > \frac{\theta^L}{\theta^H}$, then $\tilde{U}_H^H = 0$ and $\tilde{V}_H = \frac{\beta q^H}{Q_H}\theta^H$, and the constraint in (57) becomes $d_L \leq 1+(1-\phi)\delta_C - r_H\delta_C$. If d_L satisfies this constraint with strict inequality then $q^L = 0$ and the objective in (57) is increasing in d_L, because $\theta^L - Q_H\theta^H = \theta^L - \beta q^H\theta^H \geq \theta^L - \beta\theta^H > 0$. Hence $d_L^{**} = 1$ is optimal if $r_H \leq 1 - \phi$ and $d_L^{**} = 1 + (1 - \phi)\delta_C - r_H\delta_C$ is optimal if $r_H > 1 - \phi$. Consider these two alternatives in turn. If the supplier chooses $r_H \leq 1-\phi$, substitution of $q^L = 0$ and $d_L = 1$ into (57) gives the following unconstrained program:

$$\max_{q^H > 0, r_H \leq 1-\phi} \theta^L + (1 - \beta q^H)\phi\delta_S\theta^L + \beta q^H[r_H\phi\delta_S\theta^H + (1 - r_H)\delta_C\theta^H - \phi\delta_C\Delta\theta]. \tag{60}$$

From $\phi \geq \frac{\delta_C}{\delta_S}$ it follows that it is optimal to set $r_H^{**} = 1 - \phi$. For $q^H \to 0$ the remaining objective takes the value V^{RR} whereas for $q^H = 1$ the payoff becomes

$$V^{RM} = (1 + \phi\delta_S)\theta^L + \beta\phi[(1 - \phi)\delta_S\theta^H - (\delta_S - \delta_C)\theta^L]. \tag{61}$$

The corresponding menu of contracts $\{(d_L^{**}, r_L^{**}), (d_H^{**}, r_H^{**})\} = \{(1, 1), (1, 1 - \phi)\}$ rents to the low type but mixes between renting and selling to the high type. Alternatively, if the supplier chooses $r_H > 1 - \phi$ then substitution of $d_L^{**} = 1 + (1 - \phi)\delta_C - r_H\delta_C$ into (57) simplifies the program to

$$\max_{q^L, q^H, r_H > 1-\phi} \quad [1 + (1-\phi)\delta_C + \phi\delta_S - r_H\delta_C]\theta^L + \beta q^H r_H \phi\delta_S\theta^H \qquad (62)$$
$$- \quad [\beta q^H + (1-\beta)q^L](\delta_S - \delta_C)\phi\theta^L.$$

Note that the objective in (62) is decreasing in q^L and supermodular in (q^H, r_H). Hence it is optimal to set $q^L = 0$ and to choose either $q^H \to 0$ and $r_H = 1 - \phi$ or $q^H = 1$ and $r_H = 1$. The first possibility again results in the payoff V^{RR}. The second possibility gives the payoff

$$V^{rR} = (1 + \phi\delta_S)\theta^L + \phi[\beta\delta_S\Delta\theta - (1-\beta)\delta_C\theta^L] \qquad (63)$$

and corresponds to the menu $\{(d_L, r_L), (d_H, r_H)\} = \{(1 - \phi\delta_C, 1), (1, 1)\}$ which rents to both types but separates by delivering the product to the low type with a lower probability than to the high type.

A straight forward comparison of the payoff from the optimal non-learning menu, V^{RR}, with the payoffs V^{RM} and V^{rR} of the two candidates for the optimal learning menu completes our characterization of the supplier's optimal mechanism for the case where $\phi \geq \frac{\delta_C}{\delta_S}$. □

Proof of Proposition 6. The proof consists of two steps. In the first step we determine the optimal separating menu inducing posteriors $(\tilde{\beta}_L, \tilde{\beta}_H) = (0, 1)$. This allows us to show that for $\beta > \frac{\theta^L}{\theta^H}$ screening by mode of trade is dominated by ordinary, intertemporal screening. The second step compares the payoffs of the optimal separating menu with the payoffs from semi-separation $(\tilde{\beta}_L, \tilde{\beta}_H) = (\frac{\theta^L}{\theta^H}, 1)$ and pooling $(\tilde{\beta}_L, \tilde{\beta}_H) = (\beta, \beta)$. As this comparison leads to results that are well known from the literature, it is omitted. Details are available on request.

Focusing on menus that induce (full) separation $(\tilde{\beta}_L, \tilde{\beta}_H) = (0, 1)$ allows us to set $q^L = 0$, $q^H = 1$, and $\tilde{U}_L^H = \Delta\theta$, $\tilde{V}_L = \theta^L$, $\tilde{U}_H^H = 0$, $\tilde{V}_H = \theta^H$. Substitution of these values together with $d_H^{**} = 1$ from Lemma 3 into the reduced program leaves us with the following problem:

$$\max_{(d_L, r_L, r_H)} d_L\{\theta^L - \beta\theta^H - (1 - r_L)[\phi(1 - \beta)(\delta_S - \delta_C)\theta^L + (1 - \phi)\delta_C(\beta\theta^H - \theta^L)]\} \quad (64)$$
$$+\phi\delta_S\theta^L + \beta\{[1 + \delta_C(1 - r_H)]\theta^H - \phi(\delta_C\Delta\theta + \delta_S\theta^L - r_H\delta_S\theta^H)\}$$
$$\text{s.t.} \quad 1 + (1 - r_H)\delta_C - d_L[1 + (1 - r_L)\delta_C(1 - \phi)] \geq \phi\delta_C.$$

As $\beta\theta^H > \theta^L$, a decrease in d_L raises the objective while relaxing the constraint. Hence the optimal separating menu must set $d_L^{**} = 0$, i.e. it must exclude the low type. Screening by mode of trade $(d_L, r_L) = (1, 1)$ is dominated by ordinary screening because the supplier's prior is such that serving only the high type would be optimal in a static setting. □

Proof of Proposition 5. As for $\beta \geq \frac{\theta^L}{\theta^H}$ or $\phi < \frac{\delta_C}{\delta_S}$ the (fully) optimal menu refrains from randomization, we can restrict attention to the case where $\beta < \frac{\theta^L}{\theta^H}$ and $\phi \geq \frac{\delta_C}{\delta_S}$. Note first that the menu $\{(d_L, r_L), (d_H, r_H)\} = \{(0, 0), (1, 0)\}$ which sells to H while excluding L is dominated by the menu $\{(0, 0), (1, 1)\}$ which rents to the high type while excluding the low type. Similarly, the menu $\{(1, 0), (1, 0)\}$ which sells to both types is dominated by the menu $\{(1, 1), (1, 1)\}$ which rents to both types. The simple reason is that, for $\phi \geq \frac{\delta_C}{\delta_S}$ the supplier has an inherent preference for renting over selling. Further note that for the menu $\{(1, 0), (1, 1)\}$ which rents to the high type while selling to the low type, the constraint (DMC) becomes $\frac{\phi\tilde{U}_H^H}{\Delta\theta} \geq 1$ and cannot be satisfied. In other words, the menu's allocation is not implementable. Given that $d_H^{**} = 1$, the remaining candidates for the supplier's optimal deterministic menu are: $\{(1, 1), (1, 0)\}$ (selling to the high type while renting to the low type) with payoff V^{RS} given by (56); $\{(1, 1), (1, 1)\}$ (renting to both types) with payoff V^{RR} given by (59); and finally $\{(0, 0), (1, 1)\}$ (renting to the high type while excluding the low type) which results in the payoff

$$V^{ER} = \phi\delta_S\theta^L + \beta\{\theta^H + \phi(\delta_S - \delta_C)\Delta\theta\}. \quad (65)$$

Note that

$$V^{RR} < V^{RS} \Leftrightarrow \phi < \frac{\delta_C \theta^H}{\delta_C \Delta\theta + \delta_S \theta^L} \quad \text{and} \quad V^{ER} < V^{RS} \Leftrightarrow \phi < \frac{\theta^L - \beta\theta^H(1-\delta_C)}{\beta\delta_S\theta^H}. \tag{66}$$

Hence separation by mode of trade is optimal if and only if $\phi \in [\underline{\phi}, \bar{\phi}^{PP}(\beta)]$ with $\bar{\phi}^{PP}(\beta)$ given by (22). The threshold $\bar{\phi}^{PP}(\beta)$ is decreasing in β and it is straight forward to show that $\bar{\phi}^{PP}(\beta) < \bar{\phi}$. Finally, for $\phi > \bar{\phi}^{PP}(\beta)$ it remains to compare renting to both types with renting to only the high type:

$$V^{ER} < V^{RR} \Leftrightarrow \beta < \beta^{RPP}(\phi) \equiv \frac{\theta^L}{\theta^H + \phi(\delta_S - \delta_C)\Delta\theta}. \tag{67}$$

The threshold $\beta^{RPP}(\phi)$ is decreasing and converges to

$$\beta^{RPP}(1) = \frac{\theta^L}{\theta^H + (\delta_S - \delta_C)\Delta\theta} > \frac{\delta_C\theta^L}{\delta_C\theta^L + \delta_S\Delta\theta} = \underline{\beta}. \tag{68}$$

The prices specified in Proposition 5 can be determined from (11) and (12) via substitution of the corresponding allocations $\{(d_L, r_L), (d_H, r_H)\}$. □

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